Theory of the Elementary Particles

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We use lattice theory in order to determine the rest masses of the stable mesons and baryons and their spin. We can also explain the strength of the weak force which holds the lattice of the particles together, and the strong force between two elementary particles. And with the same method we can determine the mass of the muon and the electron and the mass of the electron neutrino, the muon neutrino and the tau neutrino. It turns out that the mass of the electron neutrino is equal to the mass of the muon neutrino times the fine structure constant. Only photons, neutrinos and charge are needed to explain the masses of the elementary particles.

Keywords: particle theory, particle masses, neutrino masses, spin.

Introduction

After 30 years of effort the so-called "Standard Model" of the elementary particles has not come up with a precise theoretical determination of the masses of either the mesons and baryons or of the leptons, which means that neither the mass of the fundamental electron nor the mass of the fundamental proton have been explained. The quarks, which have been introduced by Gell-Mann [1] forty years ago, are said to explain the mesons and baryons. But the standard model does not explain neither the mass, nor the charge, nor the spin of the baryons and leptons, the three fundamental time-independent properties of the particles. The fractional electric charges imposed on the quarks do not explain the charge $e^{\pm}$, neither does the spin imposed on the quarks explain the spin. The measured values of the properties of the particles are given in the Review of Particle Physics [2]. There are many other attempts to explain the elementary particles or only one of the particles, too many to list them here. For example Skyrme [3] has proposed a unified theory of the mesons and baryons, and El Naschie has, in the last years, proposed a topological theory for high energy particles and the spectrum of the quarks [4-7].

The need for the present investigation has been expressed by Feynman [8] as follows: "There remains one especially unsatisfactory feature: the observed masses of the particles, m. There is no theory that adequately explains these numbers. We use the numbers in all our theories, but we do not understand them - what they are, or where they come from. I believe that from a fundamental point of view, this is a very interesting and serious problem". Today, twenty years later, we still rely on the standard model for an explanation of the masses of the elementary particles but have not succeeded. It is time to try something different.

1 The spectrum of the masses of the particles

As we have done before [9] we will focus attention on the so-called "stable" mesons and baryons whose masses are reproduced with other data in Tables 1 and 2. It is obvious that any attempt to explain the masses of the mesons and baryons should begin with the particles that are affected by the fewest parameters. These are certainly the particles without isospin (I = 0) and without spin (J = 0), but also with strangeness S = 0, and charm C = 0. Looking at the particles with I,J,S,C = 0 it is startling to find that their masses are quite close to integer multiples of the mass of the π^0 meson. It is $\mathrm{m}(\eta) = (1.0140 \pm 0.0003) \cdot 4\mathrm{m}(\pi^0)$, and the mass of the resonance η' is $\mathrm{m}(\eta') = (1.0137 \pm 0.00015) \cdot 7\mathrm{m}(\pi^0)$. Three particles seem hardly to be sufficient to establish a rule. However, if we look a little further we find that $\mathrm{m}(\Lambda) = 1.0332 \cdot 8\mathrm{m}(\pi^0)$ or $\mathrm{m}(\Lambda) = 1.0190 \cdot 2\mathrm{m}(\eta)$. We note that the Λ baryon has spin 1/2, not spin 0 as the π^0, η mesons. Nevertheless, the mass of Λ is close to $8\mathrm{m}(\pi^0)$. Furthermore we have $\mathrm{m}(\Sigma^0) = 0.9817 \cdot 9\mathrm{m}(\pi^0)$, $\mathrm{m}(\Xi^0) = 0.9742 \cdot 10\mathrm{m}(\pi^0)$, $\mathrm{m}(\Omega^-) = 1.0325 \cdot 12\mathrm{m}(\pi^0) = 1.0183 \cdot 3\mathrm{m}(\eta)$, ($\Omega^-$ is charged and has spin 3/2). Finally the masses of the charmed baryons are $\mathrm{m}(\Lambda_c^+) = 0.99645 \cdot 17\mathrm{m}(\pi^0) = 1.024 \cdot 2\mathrm{m}(\Lambda)$, $\mathrm{m}(\Sigma_c^0) = 1.00995 \cdot 18\mathrm{m}(\pi^0)$, $\mathrm{m}(\Xi_c^0) = 1.0170 \cdot 18\mathrm{m}(\pi^0)$, and $\mathrm{m}(\Omega_c^0) = 0.99925 \cdot 20\mathrm{m}(\pi^0)$.

Now we have enough material to formulate the *integer multiple rule* of the particle masses, according to which the masses of the $\eta, \Lambda, \Sigma^0, \Xi^0, \Omega^-, \Lambda_c^+, \Sigma_c^0$, Ξ_c^0, and Ω_c^0 particles are, in a first approximation, integer multiples of the mass of the π^0 meson, although some of the particles have spin, and may also have charge as well as strangeness and charm. A consequence of the integer multiple rule must be that the ratio of the mass of any meson or baryon listed above divided by the mass of another meson or baryon listed above is equal

Table 1: The ratios $m/m(\pi^0)$ of the π^0 and η mesons and of the baryons of the γ-branch.

	$m/m(\pi^0)$	multiples	decays	fraction (%)	spin	mode
π^0	1.0000	$1.0000 \cdot \pi^0$	$\gamma\gamma$ $e^+e^-\gamma$	98.798 1.198	0	(1.)
η	4.0563	$1.0141 \cdot 4\pi^0$	$\gamma\gamma$ $3\pi^0$ $\pi^+\pi^-\pi^0$ $\pi^+\pi^-\gamma$	39.43 32.51 22.6 4.68	0	(2.)
Λ	8.26575	$1.0332 \cdot 8\pi^0$ $1.0190 \cdot 2\eta$	$p\pi^-$ $n\pi^0$	63.9 35.8	$\frac{1}{2}$	2∗(2.)
Σ^0	8.8359	$0.9817 \cdot 9\pi^0$	$\Lambda\gamma$	100	$\frac{1}{2}$	2∗(2.) + (1.)
Ξ^0	9.7412	$0.9741 \cdot 10\pi^0$	$\Lambda\pi^0$	99.52	$\frac{1}{2}$	2∗(2.) + 2(1.)
Ω^-	12.390	$1.0326 \cdot 12\pi^0$ $1.0183 \cdot 3\eta$	ΛK^- $\Xi^0\pi^-$ $\Xi^-\pi^0$	67.8 23.6 8.6	$\frac{3}{2}$	3∗(2.)
Λ_c^+	16.939	$0.99645 \cdot 17\pi^0$ $0.9630 \cdot 17\pi^\pm$	many		$\frac{1}{2}$	2∗(2.) + (3.)
Σ_c^0	18.179	$1.0099 \cdot 18\pi^0$	$\Lambda_c^+\pi^-$	≈100	$\frac{1}{2}$	$\Lambda_c^+ + \pi^-$
Ξ_c^0	18.307	$1.0170 \cdot 18\pi^0$	eleven	(seen)	$\frac{1}{2}$	2∗(3.)
Ω_c^0	19.985	$0.99925 \cdot 20\pi^0$ $0.9854 \cdot 5\eta$	seven	(seen)	$\frac{1}{2}$	2∗(3.) + 2(1.)

[1]The modes apply to neutral particles only. The ∗ marks coupled modes.

to the ratio of two integer numbers. And indeed, for example $m(\eta)/m(\pi^0)$ is practically two times (exactly $0.9950 \cdot 2$) the ratio $m(\Lambda)/m(\eta)$. There is also the ratio $m(\Omega^-)/m(\Lambda) = 0.9993 \cdot 3/2$. We have furthermore e.g. the ratios $m(\Lambda)/m(\eta) = 1.019 \cdot 2$, $m(\Omega^-)/m(\eta) = 1.018 \cdot 3$, $m(\Lambda_c^+)/m(\Lambda) = 1.0247 \cdot 2$, $m(\Sigma_c^0)/m(\Sigma^0) = 1.0287 \cdot 2$, $m(\Omega_c^0)/m(\Xi^0) = 1.0258 \cdot 2$, and $m(\Omega_c^0)/m(\eta) = 0.9854 \cdot 5$.

We will call, for reasons to be explained soon, the particles discussed above, which follow in a first approximation the integer multiple rule, the *γ-branch* of the particle spectrum. The mass ratios of these particles are in Table 1. The deviation of the mass ratios from exact integer multiples of $m(\pi^0)$ is at most 3.3%, the average of the factors before the integer multiples of $m(\pi^0)$ of the nine γ-branch particles in Table 1 is 1.0066 ± 0.0184. From a least square analysis follows that the masses of the ten particles on Table 1 lie on a straight line given by the formula

$$m(N)/m(\pi^0) = 1.0065\,N - 0.0043 \qquad N > 1, \tag{1}$$

where N is the integer number nearest to the actual ratio of the particle mass divided by $m(\pi^0)$. The correlation coefficient in Eq.(1) has the nearly perfect value $R^2 = 0.999$.

The integer multiple rule applies to more than just the stable mesons and baryons. The integer multiple rule applies also to the γ-branch baryon resonances which have spin J = 1/2 and the meson resonances with I,J $\leq$ 1, listed in [2] or in Table 3 of Appendix A. The Ω^- baryon will not be considered because it has spin 3/2 but would not change the following equation significantly. If we consider all mesons and baryons of the γ-branch in Table 3, “stable” or unstable, then we obtain from a least square analysis the formula

$$m(N)/m(\pi^0) = 0.999\,N + 0.0867 \qquad N > 1, \tag{2}$$

with the correlation coefficient 0.9999. The line through the points is shown in Fig. 1 which tells that 41 particles of the γ-branch of different spin and isospin, strangeness and charm; five I,J = 0,0 η mesons, fifteen J = 1/2 baryons, ten I = 0, J = 0,1 $c\bar{c}$ mesons, ten I = 0, J = 0,1 $b\bar{b}$ mesons and the π^0 meson with I,J = 1,0, lie on a straight line with slope 0.999. In other words they approximate the integer multiple rule very well. Spin 1/2 and spin 1 does not seem to affect the integer multiple rule, i.e. the ratios of the particle masses, neither does strangeness S $\neq$ 0 and charm C $\neq$ 0.

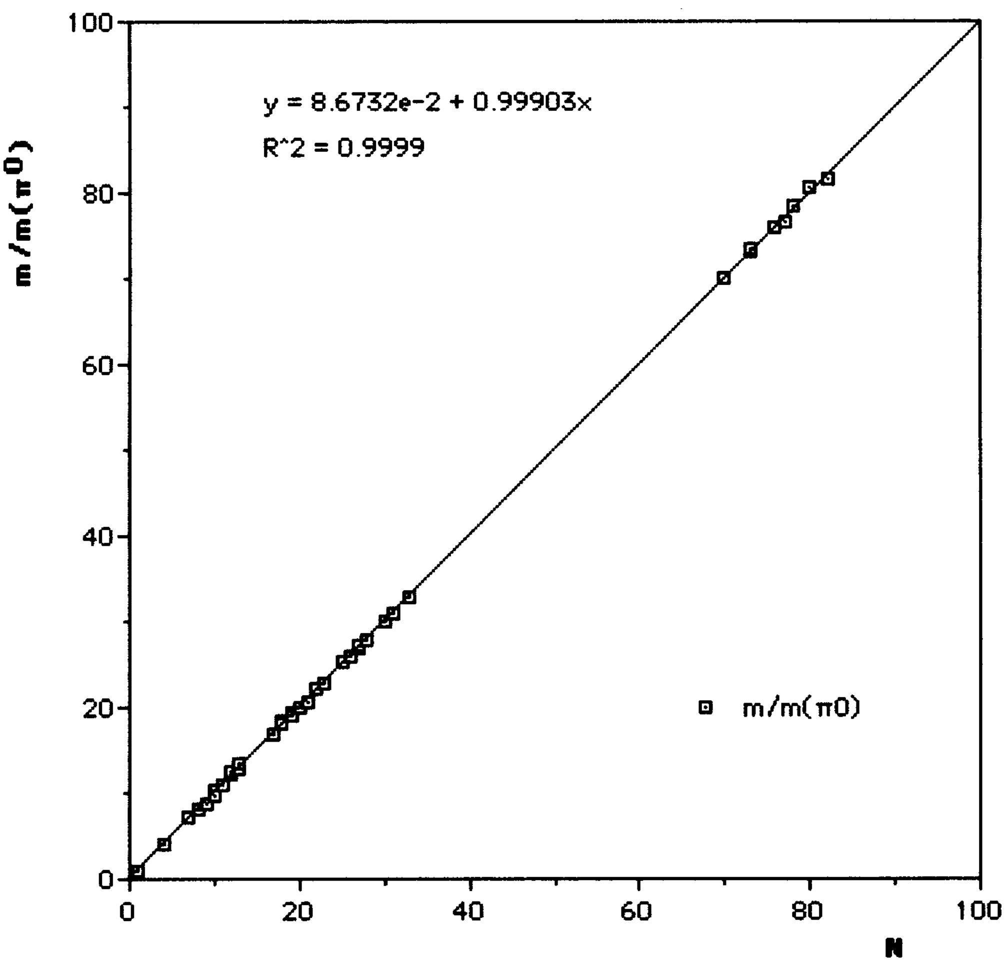

Fig. 1: The mass of the mesons and baryons of the γ-branch, stable or unstable, with $\mathrm{I} \leq 1, \mathrm{J} \leq 1$ in units of $\mathrm{m}(\pi^0)$ as a function of the integer N, demonstrating the integer multiple rule.

Searching for what else the $\pi^0, \eta, \Lambda, \Sigma^0, \Xi^0, \Omega^-$ particles have in common, we find that the principal decays (decays with a fraction $>1\%$) of these particles, as listed in Table 1, involve primarily γ-rays, the characteristic case is $\pi^0 \rightarrow \gamma\gamma$ (98.8%). We will later on discuss a possible explanation for the 1.198% of the decays of π^0 which do not follow the $\gamma\gamma$ route but decay via $\pi^0 \rightarrow e^+ + e^- + \gamma$. After the γ-rays the next most frequent decay product of the heavier particles of the γ-branch are π^0 mesons which again decay into $\gamma\gamma$. To describe the decays in another way, the principal decays of the particles listed above take place *always without the emission of neutrinos*; see Table 1. There the decays and the fractions of the principal decay modes are given, taken from the Review of Particle Physics. We cannot consider decays with fractions $<1\%$. We will refer to the particles whose masses are approximately integer multiples of the mass of the π^0 meson, and which decay without the emission of neutrinos, as the γ-*branch* of the particle spectrum.

To summarize the facts concerning the γ-branch. Within 0.66% on the average the masses of the stable particles of the γ-branch in Table 1 are integer multiples (namely $4, 8, 9, 10, 12$, and even $17, 18, 20$) of the mass of the π^0 meson. It is improbable that nine particles have masses so close to integer multiples of $m(\pi^0)$ if there is no correlation between them and the π^0 meson. It has, on the other hand, been argued that the integer multiple rule is a numerical coincidence. But the probability that the mass ratios of nine particles of the γ-branch fall by coincidence on integer numbers between 1 and 20 instead on all possible numbers between 1 and 20 with two decimals after the period is smaller than 10^{-20}, i.e. nonexistent. The integer multiple rule is not affected by more than 3% by the spin, the isospin, the strangeness, and by charm. The integer multiple rule seems even to apply to the Ω^- and Λ_c^+ particles, although they are charged. In order for the integer multiple rule to be valid the deviation of the ratio $m/m(\pi^0)$ from an integer number must be smaller than 1/2N, where N is the integer number closest to the actual ratio $m/m(\pi^0)$. That means that the permissible deviation decreases rapidly with increased N. All particles of the γ-branch have deviations smaller than 1/2N.

The remainder of the stable mesons and baryons are the $\pi^\pm$, $K^{\pm,0}$, p, n, $D^{\pm,0}$, and $D_s^\pm$ particles which make up the neutrino-branch (ν-branch) of the particle spectrum. The ratios of their masses are given in Table 2.

These particles are in general charged, exempting the K^0 and D^0 mesons and the neutron n, in contrast to the particles of the γ-branch, which are in general neutral. It does not make a significant difference whether one

Table 2: The ν-branch of the particle spectrum

	m/m($\pi^{\pm}$)	multiples	decays	fraction (%)	spin	mode
$\pi^{\pm}$	1.0000	$1.0000 \cdot \pi^{\pm}$	$\mu^{+}\nu_{\mu}$	99.9877	0	(1.)
$K^{\pm,0}$	3.53712	$0.8843 \cdot 4\pi^{\pm}$	$\mu^{+}\nu_{\mu}$	63.43	0	(2.) + π^{0}
			$\pi^{\pm}\pi^{0}$	21.13		($K^{\pm}$)
			$\pi^{+}\pi^{-}\pi^{+}$	5.58		(2.) + $\pi^{\mp}$
			$\pi^{0}e^{+}\nu_{e}$ (K^{+}_{e3})	4.87		(K^{0},$\overline{K^{0}}$)
			$\pi^{0}\mu^{+}\nu_{\mu}$ ($K^{+}_{\mu 3}$)	3.27		
n	6.73185	$0.8415 \cdot 8\pi^{\pm}$	$p\,e^{-}\overline{\nu}_{e}$	100.	$\frac{1}{2}$	2∗(2.)
		$0.9516 \cdot 2K^{\pm}$				+ $2\pi^{\pm}$
		$0.9439 \cdot (K^{0} + \overline{K}^{0})$				
$D^{\pm,0}$	13.393	$0.8370 \cdot 16\pi^{\pm}$	e^{+} anything	17.2	0	2(2∗(2.)
		$0.9466 \cdot 4K^{\pm}$	K^{-} anything	24.2		+ $2\pi^{\pm}$)
		$0.9954 \cdot (p + \bar{n})$	$\overline{K}^{0}$ anything			
			+ K^{0} anything	59		
			η anything	< 13		
			K^{+} anything	5.8		
$D^{\pm}_{s}$	14.102	$0.8295 \cdot 17\pi^{\pm}$	K^{-} anything	13	0	body
		$0.9967 \cdot 4K^{\pm}$	$\overline{K}^{0}$ anything			centered
			+ K^{0} anything	39		cubic
			K^{+} anything	20		
			e^{+} anything	8		

[2]The particles with negative charges have conjugate charges of the listed decays. Only the decays of $K^{\pm}$ and $D^{\pm}$ are listed. The oscillation modes carry one electric charge. The ∗ marks coupled modes.

considers the mass of a particular charged or neutral particle. After the π mesons, the largest mass difference between charged and neutral particles is that of the K mesons (0.81%), and thereafter all mass differences between charged and neutral particles are $< 0.5\%$. The integer multiple rule does not immediately apply to the masses of the ν-branch particles if $\mathrm{m}(\pi^{\pm})$ (or $\mathrm{m}(\pi^0)$) is used as reference, because $\mathrm{m}(\mathrm{K}^{\pm}) = 0.8843 \cdot 4\mathrm{m}(\pi^{\pm})$. $0.8843 \cdot 4 = 3.537$ is far from integer. Since the masses of the π^0 meson and the $\pi^{\pm}$ mesons differ by only 3.4% it has been argued that the $\pi^{\pm}$ mesons are, but for the isospin, the same particles as the π^0 meson, and that therefore the $\pi^{\pm}$ mesons cannot start another particle branch. However, this argument is not supported by the completely different decays of the π^0 mesons and the $\pi^{\pm}$ mesons. The π^0 meson decays almost exclusively into $\gamma\gamma$ (98.8%), whereas the $\pi^{\pm}$ mesons decay practically exclusively into μ mesons and neutrinos, as in $\pi^+ \rightarrow \mu^+ + \nu_\mu$ (99.9877%). Furthermore, the lifetimes of the π^0 and the $\pi^{\pm}$ mesons differ by nine orders of magnitude, being $\tau(\pi^0) = 8.4 \cdot 10^{-17}$ sec versus $\tau(\pi^{\pm}) = 2.6 \cdot 10^{-8}$ sec.

If we make the $\pi^{\pm}$ mesons the reference particles of the ν-branch, then we must multiply the mass ratios $\mathrm{m}/\mathrm{m}(\pi^{\pm})$ of the above listed particles with an average factor 0.848 ± 0.025, as follows from the mass ratios on Table 2. The integer multiple rule may, however, apply directly if one makes $\mathrm{m}(\mathrm{K}^{\pm})$ the reference for masses larger than $\mathrm{m}(\mathrm{K}^{\pm})$. The mass of the neutron is $0.9516 \cdot 2\mathrm{m}(\mathrm{K}^{\pm})$, which is only a fair approximation to an integer multiple. There are, on the other hand, outright integer multiples in $\mathrm{m}(\mathrm{D}^{\pm}) = 0.9954 \cdot (\mathrm{m(p)} + \mathrm{m}(\bar{\mathrm{n}}))$, and in $\mathrm{m}(\mathrm{D}_s^{\pm}) = 0.9968 \cdot 4\mathrm{m}(\mathrm{K}^{\pm})$. A least square analysis of the masses of the ν-branch in Table 2 yields the formula

$$\mathrm{m}(N)/0.853\mathrm{m}(\pi^{\pm}) = 1.000\,N + 0.00575 \qquad N > 1, \tag{3}$$

with $\mathrm{R}^2 = 0.998$. This means that the particles of the ν-branch are integer multiples of $\mathrm{m}(\pi^{\pm})$ times the factor 0.853. One must, however, consider that the $\pi^{\pm}$ mesons are not necessarily the perfect reference for all ν-branch particles, because $\pi^{\pm}$ has I = 1, whereas for example $\mathrm{K}^{\pm}$ has I = 1/2 and S = ± 1 and the neutron has also I = 1/2. Actually the factor 0.853 in Eq.(3) is only an average. The mass ratios indicate that this factor decreases slowly with increased m(N). The existence of the factor and its decrease will be explained later.

Contrary to the particles of the γ-branch, the ν-branch particles decay preferentially with the emission of neutrinos, the foremost example is $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ with a fraction of 99.9877%. Neutrinos characterize the weak interaction. We will refer to the particles in Table 2 as the *neutrino branch* (ν-branch) of the particle spectrum. We emphasize that a weak decay of the particles of the ν-branch is by no means guaranteed. Although the neutron decays via $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^{-} + \bar{\nu}_{e}$ in 887 sec (100%), the proton is stable. There are, on the other hand, weak decays such as e.g. $\mathrm{K}^{+} \rightarrow \pi^{+}\pi^{-}\pi^{+}$ (5.59%), but the subsequent decays of the $\pi^{\pm}$ mesons lead to neutrinos and $\mathrm{e}^{\pm}$.

To summarize the facts concerning the ν-branch of the mesons and baryons. The masses of these particles seem to follow the integer multiple rule if one uses the $\pi^{\pm}$ mesons as reference, however the mass ratios share a common factor 0.85 ± 0.025.

To summarize what we have learned about the *integer multiple rule*: In spite of differences in charge, spin, strangeness, and charm the masses of the "stable" mesons and baryons of the γ-branch are integer multiples of the mass of the π^{0} meson within at most 3.3% and on the average within 0.66%. Correspondingly, the masses of the "stable" particles of the ν-branch are, after multiplication with a factor 0.85 ± 0.025, integer multiples of the mass of the $\pi^{\pm}$ mesons. The integer multiple rule has been anticipated much earlier by Nambu [10], who wrote in 1952 that "some regularity [in the masses of the particles] might be found if the masses were measured in a unit of the order of the π-meson mass". A similar suggestion has been made by Fröhlich [11]. The integer multiple rule suggests that the particles are the result of superpositions of modes and higher modes of a wave equation.

2 Standing waves in a cubic lattice

We will now study, as we have done in [12], whether the so-called "stable" particles of the γ-branch cannot be described by the frequency spectrum of standing waves in a cubic lattice, which can accommodate automatically the Fourier frequency spectrum of an extreme short-time collision by which the particles are created. The investigation of the consequences of lattices for particle theory was initiated by Wilson [13] who studied a cubic fermion lattice. His study has developed over time into lattice QCD.

It will be necessary for the following to outline the most elementary aspects of the theory of lattice oscillations. The classic paper describing lattice

oscillations is from Born and v. Karman [14], henceforth referred to as B&K. They looked at first at the oscillations of a one-dimensional chain of points with mass m, separated by a constant distance a. This is the *monatomic* case, all lattice points have the same mass. B&K assume that the forces exerted on each point of the chain originate only from the two neighboring points. These forces are opposed to and proportional to the displacements, as with elastic springs (Hooke's law). The equation of motion is in this case

$$\mathrm{m}\ddot{u}_n = \alpha(u_{n+1} - u_n) - \alpha(u_n - u_{n-1})\,. \tag{4}$$

The u_n are the displacements of the mass points from their equilibrium position which are apart by the distance a. The dots signify, as usual, differentiation with respect to time, α is a constant characterizing the force between the lattice points, and n is an integer number. For $a \to 0$ Eq.(4) becomes the wave equation $\mathrm{c}^2\partial^2\mathrm{u}/\partial\mathrm{x}^2 = \partial^2\mathrm{u}/\partial\mathrm{t}^2$ (B&K).

In order to solve Eq.(4) B&K set

$$u_n = Ae^{i(\omega\, t + n\phi)}\,, \tag{5}$$

which is obviously a temporally and spatially periodic solution or describes *standing waves.* n is an integer, with n < N, where N is the number of points in the chain. $\phi = 0$ is the monochromatic case. We also consider higher modes, by replacing nϕ in Eq.(5) by n$'\phi$, where n$'$ is n times an integer number > 1. The wavelengths are then shorter by one over the integer number n$'$/n. At n$\phi = \pi/2$ are nodes, where for all times t the displacements are zero, as with standing waves $\mathrm{f(x,t)} = \mathrm{A}\cos(\omega\mathrm{t})\cos(\mathrm{n}\phi) = \mathrm{A}\cos(\omega\mathrm{t})\cos(\mathrm{kx})$. If a displacement is repeated after n points we have n$a = \lambda$, where λ is the wavelength, a the lattice constant, and it must be n$\phi = 2\pi$ according to (5). It follows that

$$\lambda = 2\pi\, a/\phi\,. \tag{6}$$

Inserting (5) into (4) one obtains a continuous frequency spectrum of the standing waves as given by Eq.(5) of B&K

$$\omega = \pm\, 2\sqrt{\alpha/\mathrm{m}}\,\sin(\phi/2)\,. \tag{7}$$

B&K point out that there is not only a continuum of frequencies, but also a *maximal frequency* which is reached when $\phi = \pi$, or at the minimum of the possible wavelengths $\lambda = 2a$. The boundary conditions are periodic, that

means that $u_n = u_{n+N}$, where N is the number of points in the chain. Born referred to the periodic boundary condition as a "mathematical convenience". The number of normal modes must be equal to the number of particles in the lattice.

Born's model of the crystals has been verified in great detail by X-ray scattering and even in much more complicated cases by neutron scattering. The theory of lattice oscillations has been pursued in particular by Blackman [15], a summary of his and other studies is in [16]. Comprehensive reviews of the results of linear studies of lattice dynamics have been written by Born and Huang [17], by Maradudin et al. [18], and by Ghatak and Kothari [19].

3 The masses of the γ-branch particles

We will now assume, as seems to be quite natural, that the particles *consist of the same particles into which they decay*, directly or ultimately. We know this from atoms, which consist of nuclei and electrons, and from nuclei, which consist of protons and neutrons. Quarks have never been observed among the decay products of elementary particles. For the γ-branch particles our assumption means that they consist of photons. Photons and π^0 mesons are the principal decay products of the γ-branch particles, the characteristic example is $\pi^0 \rightarrow \gamma\gamma$ (98.8%). Table 1 shows that there are decays of the γ-branch particles which lead to particles of the ν-branch, in particular to pairs of π^+ and π^- mesons. It appears that this has to do with pair production in the γ-branch particles. Pair production is evident in the decay $\pi^0 \rightarrow e^+ + e^- + \gamma$ (1.198%) or in the π^0 meson's third most frequent decay $\pi^0 \rightarrow e^+e^-e^+e^-$ ($3.14 \cdot 10^{-3}$%). Pair production requires the presence of electromagnetic waves of high energy. Anyway, the explanation of the γ-branch particles must begin with the explanation of the most simple example of its kind, the π^0 meson, which by all means seems to consist of photons. The composition of the particles of the γ-branch suggested here offers a direct route from the formation of a γ-branch particle, through its lifetime, to its decay products. Particles that are made of photons are necessarily neutral, as the majority of the particles of the γ-branch are.

We also base our assumption that the particles of the γ-branch are made of photons on the circumstances of the formation of the γ-branch particles. The most simple and straightforward creation of a γ-branch particle are the reactions $\gamma + p \rightarrow \pi^0 + p$, or in the case that the spins of γ and p are parallel

$\gamma + \mathrm{p} \to \pi^0 + \mathrm{p} + \gamma'$. A photon impinges on a proton and creates a π^0 meson. The considerations which follow apply as well for other photoproductions such as $\gamma + \mathrm{p} \to \eta + \mathrm{p}$ or $\gamma + \mathrm{d} \to \pi^0 + \mathrm{d}$ and to the photoproduction of Λ in $\gamma + \mathrm{p} \to \Lambda + \mathrm{K}^+$, but also for the electroproductions $\mathrm{e}^- + \mathrm{p} \to \pi^0 + \mathrm{e}^- + \mathrm{p}$ or $\mathrm{e}^- + \mathrm{d} \to \pi^0 + \mathrm{e}^- + \mathrm{d}$, see Rekalo et al. [20]. The most simple example of the creation of a γ-branch particle by a strong interaction is the reaction $\mathrm{p} + \mathrm{p} \to \mathrm{p} + \mathrm{p} + \pi^0$. The electromagnetic energy accumulated in a proton during its acceleration reappears as the π^0 meson.

In $\gamma + \mathrm{p} \to \pi^0 + \mathrm{p}$ the pulse of the incoming electromagnetic wave is in 10^{-23} sec converted into a continuum of electromagnetic waves with frequencies ranging from 10^{23} sec^{-1} to $\nu \to \infty$ according to Fourier analysis. There must be a cutoff frequency, otherwise the energy in the sum of the frequencies would exceed the energy of the incoming electromagnetic wave. The wave packet so created decays, according to experience, after $8.4 \cdot 10^{-17}$ sec into two electromagnetic waves or γ-rays. It seems to be very unlikely that Fourier analysis does not hold for the case of an electromagnetic wave impinging on a proton. The question then arises of what happens to the electromagnetic waves in the timespan of 10^{-16} seconds between the creation of the wave packet and its decay into two γ-rays? We will show that the electromagnetic waves can continue to exist for the 10^{-16} seconds until the wave packet decays.

If the wave packet created by the collision of a γ-ray with a proton consists of electromagnetic waves, then the waves cannot be progressive because the wave packet must have a *rest mass*. The rest mass is the mass of a particle whose center of mass does not move. However *standing electromagnetic waves* have a rest mass. Standing electromagnetic waves are equivalent to a lattice, because in standing waves the waves travel back and forth between the nodes, just as lattice points oscillate between the nodes of the lattice oscillations. The oscillations in the lattice take care of the continuum of frequencies of the Fourier spectrum of the collision. So we assume that the very many photons in the wave packet are held together in a cubic lattice. It is not unprecedented that photons have been considered to be building blocks of the elementary particles. Schwinger [21] has once studied an exact one-dimensional quantum electrodynamical model in which the photon acquired a mass $\sim e^2$.

We will now investigate the standing waves in a cubic photon lattice. We assume that the lattice is held together by a weak force acting from one lattice point to its nearest neighbors. We assume that the range of this force is 10^{-16} cm, because the range of the weak nuclear force is on the order of

10^{-16} cm, as stated e.g. on p.25 of Perkins [22]. We set the lattice constant at

$$a = 1 \cdot 10^{-16}\,\mathrm{cm}\,, \tag{8}$$

as we have done originally in [23]. The lattice constant of a cubic lattice can be derived from lattice theory, see Appendix B. For the sake of simplicity we set the sidelength of the lattice at 10^{-13} cm, there are then 10^9 lattice points. The exact size of the nucleon is given in [26] and will be used later. As we will see the ratios of the masses of the particles are independent of the sidelength of the lattice. Because it is the most simple case, we assume that a central force acts between the lattice points. We cannot consider spin, isospin, strangeness or charm of the particles. The frequency equation for the waves in an isotropic monatomic cubic lattice with central forces is, in the one-dimensional case, given by Eq.(7). The direction of the waves is determined by the direction of the incoming γ-ray.

According to Eq.(13) of B&K the force constant α is

$$\alpha = a\,(c_{11} - c_{12} - c_{44})\,, \tag{9}$$

where c_{11}, c_{12} and c_{44} are the elastic constants in continuum mechanics which applies in the limit $a \rightarrow 0$. If we consider central forces then $\mathrm{c}_{12} = \mathrm{c}_{44}$ which is the classical Cauchy relation. Isotropy requires that $\mathrm{c}_{44} = (\mathrm{c}_{11} - \mathrm{c}_{12})/2$. The waves are longitudinal. Transverse waves in a cubic lattice with central forces are not possible according to [19]. All frequencies that solve Eq.(7) come with either a plus or a minus sign which is, as we will see, important. The reference frequency in Eq.(7) is

$$\nu_0 = \sqrt{\alpha/4\pi^2\mathrm{m}} = \mathrm{c}/2\pi\,a\,, \tag{10}$$

as we will see, using Eq.(12). c is the velocity of light.

The *limitation of the group velocity* has now to be considered. The group velocity is given by

$$\mathrm{c}_g = \frac{d\omega}{dk} = a\sqrt{\frac{\alpha}{\mathrm{m}}} \cdot \frac{df(\phi)}{d\phi}\,. \tag{11}$$

The group velocity of the photons has to be equal to the velocity of light c throughout the entire frequency spectrum, because photons move with the velocity of light. In order to learn how this requirement affects the frequency distribution we have to know the value of $\sqrt{\alpha/\mathrm{m}}$ in a photon lattice. But

we do not have information about what either α or m might be in this case. In the following we set $a\sqrt{\alpha/\mathrm{m}} = \mathrm{c}$, which means, since $a = 10^{-16}$ cm, that $\sqrt{\alpha/\mathrm{m}} = 3 \cdot 10^{26}$ sec^{-1}, or that the corresponding period is $\tau = 1/3 \cdot 10^{-26}$ sec, which is the time it takes for a wave to travel with the velocity of light over one lattice distance. With

$$\mathrm{c} = a\sqrt{\alpha/\mathrm{m}} \tag{12}$$

the equation for the group velocity is

$$\mathrm{c}_g = \mathrm{c} \cdot df/d\phi\,. \tag{13}$$

For photons that means, since c_g must then always be equal to the velocity of light c, that df/dϕ = 1. This requirement determines the form of the frequency distribution regardless of the order of the mode of oscillation or it means that instead of the sine function in Eq.(7) the frequency is given by

$$\nu = \pm\,\nu_0(\,\phi + \phi_0\,)\,. \tag{14}$$

For the time being we will disregard ϕ_0 in Eq.(14) because $\phi_0 = 0$ when the boundary condition is periodic. The frequencies of the corrected spectrum in Eq.(14) must increase from $\nu = 0$ at the origin $\phi = 0$ with slope 1 (in units of ν_0) until the maximum is reached at $\phi = \pi$. The energy contained in the oscillations must be proportional to the sum of all frequencies (Eq.15). The *second mode* of the lattice oscillations contains 4 times as much energy as the basic mode, because the frequencies are twice the frequencies of the basic mode, and there are twice as many oscillations. Adding, by superposition, to the second mode different numbers of basic modes or of second modes will give exact integer multiples of the energy of the basic mode. Now we understand the integer multiple rule of the particles of the γ-branch. There is, in the framework of this theory, on account of Eq.(14), no alternative but *integer multiples* of the basic mode for the energy contained in the frequencies of the different modes or for superpositions of different modes. In other words, the masses of the different particles are integer multiples of the mass of the π^0 meson, if there is no spin, isospin, strangeness or charm.

We remember that the measured masses in Table 1, which incorporate different spins, isospins, strangeness and charm, spell out the integer multiple rule within on the average 0.65% accuracy. It is worth noting that *there is no free parameter* if one takes the ratio of the energies contained in

the frequency distributions of the different modes, because the factor $\sqrt{\alpha/\mathrm{m}}$ in Eq.(7) or ν_0 in Eq.(14) cancels. This means, in particular, that the ratios of the frequency distributions, or the mass ratios, are independent of the mass of the photons at the lattice points, as well as of the magnitude of the force between the lattice points.

It is obvious that the integer multiples of the sum of the frequencies in the particles are only a first approximation of the theory of lattice oscillations and of the mass ratios of the particles. The equation of motion in the lattice Eq.(4) does not apply in the eight corners of the cube, nor does it apply to the twelve edges nor, in particular, to the six sides of the cube. A cube with 10^9 lattice points is not correctly described by the periodic boundary condition we have used to derive Eq.(7), but is what is referred to as a microcrystal. A phenomenological theory of the frequency distributions in microcrystals, considering in particular the surface energy, can be found in Chapter 6 of Ghatak and Kothari [19]. The surface energy may account for the small deviations of the mass ratios of the mesons and baryons from the integer multiple rule of the oscillations in a cube. However, it seems to be futile to pursue a more accurate determination of the oscillation frequencies before we know what the interaction of the electron with mass is. The mass of the electron is 0.378% of the mass of the π^0 meson and hence is a substantial part of the deviation of the mass ratios from the integer multiple rule.

Let us summarize our findings concerning the γ-branch. The particles of the γ-branch consist of standing electromagnetic waves. The π^0 meson is the basic mode. The η meson corresponds to the second mode, as is suggested by $\mathrm{m}(\eta) \approx 4\mathrm{m}(\pi^0)$. The Λ baryon corresponds to the superposition of two second modes, as is suggested by $\mathrm{m}(\Lambda) \approx 2\mathrm{m}(\eta)$. This superposition apparently results in the creation of spin 1/2. The two modes would then have to be coupled. The Σ^0 and Ξ^0 baryons are superpositions of one or two basic modes on the Λ baryon. The Ω^- particle corresponds to the superposition of three coupled second modes as is suggested by $\mathrm{m}(\Omega^-) \approx 3\mathrm{m}(\eta)$. This procedure apparently causes spin 3/2. The charmed Λ_c^+ baryon seems to be the first particle incorporating a third mode. Σ_c^0 is apparently the superposition of a negatively charged basic mode on Λ_c^+, as is suggested by the decay of Σ_c^0. The easiest explanation of Ξ_c^0 is that it is the superposition of two coupled third modes. The superposition of two modes of the same type is, as in the case of Λ, accompanied by spin 1/2. The Ω_c^0 baryon is apparently the superposition of two basic modes on the Ξ_c^0 particle. All neutral particles of the γ-branch

are thus accounted for. The explanation of the charged γ-branch particles $\Sigma^{\pm}$ and Ξ^{-} has been described in [67]. The modes of the particles are listed in Table 1.

We have also found the γ-branch *antiparticles*. The rest mass of the antiparticles follows from the sum of the energies hν in the negative frequencies which solve Eq.(7) or Eq.(14). As the particles, the antiparticles of the γ-branch consist of standing electromagnetic waves. They have the same rest mass as the particles. Antiparticles have always been associated with negative energies. Following Dirac's argument for electrons and positrons, we associate the masses with the negative frequency distributions with antiparticles. We emphasize that the existence of antiparticles is an automatic consequence of our theory.

All particles of the γ-branch are unstable with lifetimes on the order of 10^{-10} sec or shorter. Born [24] has shown that the oscillations in cubic lattices held together by central forces are unstable. It seems, however, to be possible that the particles can be unstable for reasons other than the instability of the lattice which apparently causes the most frequent (electromagnetic) decay of the π^0 meson $\pi^0 \rightarrow \gamma\gamma$ (98.798%), or the most frequent (electromagnetic) decay of the η meson $\eta \rightarrow \gamma\gamma$ (39.43%). Pair production seems to make it possible to understand the decay of the π^0 meson $\pi^0 \rightarrow e^- + e^+ + \gamma$ (1.198%). Since in our model the π^0 meson consists of a multitude of electromagnetic waves it seems that pair production takes place within the π^0 meson, and even more so in the higher modes of the γ-branch where the electrons and positrons created by pair production tend to settle on mesons, as e.g. in $\eta \rightarrow \pi^+ + \pi^- + \pi^0$ (22.6%) or in the decay $\eta \rightarrow \pi^+ + \pi^- + \gamma$ (4.68%), where the origin of the pair of charges is more apparent. Pair production is also evident in the decays $\eta \rightarrow e^+e^-\gamma$ (0.6%) or $\eta \rightarrow e^+e^-e^+e^-$ ($6.9{\cdot}10^{-3}$%).

Finally we must explain the reason for which the photon lattice or the γ-branch particles are limited in size to a particular value of about 10^{-13} cm, as the experiments show. Conventional lattice theory using the periodic boundary condition does not limit the size of a crystal, and in fact very large crystals exist. If, however, the lattice consists of standing electromagnetic waves the size of the lattice is limited by the radiation pressure. The lattice will necessarily break up at the latest when the outward directed radiation pressure is equal to the inward directed elastic force which holds the lattice together. For details we refer to [25].

4 The rest mass of the π^0 meson

So far we have studied the ratios of the masses of the particles. We will now determine the mass of the π^0 meson in order to validate that the mass ratios link with the actual masses of the particles. The energy of the π^0 meson is

$$\mathrm{E}(\mathrm{m}(\pi^0)) = 134.9766\,\mathrm{MeV} = 2.16258 \cdot 10^{-4}\,\mathrm{erg}.$$

The sum of the energies $\mathrm{E} = \mathrm{h}\nu$ of the frequencies of the one-dimensional waves in π^0 seems to be given by the equation

$$\mathrm{E}_\nu = \frac{\mathrm{Nh}\nu_0}{2\pi} \int\limits_{-\pi}^{\pi} f(\phi) d\phi\,. \tag{15}$$

N is the number of all lattice points. The total energy of the frequencies in a cubic lattice is equal to the number N of the oscillations times the average of the energy of the individual frequencies. In order to arrive at an exact value of N in Eq.(15) we have to use the correct value of the radius of the proton, for which we use, as in [9],

$$r_p = (0.880 \pm 0.015) \cdot 10^{-13}\,\mathrm{cm}\,, \tag{16}$$

according to [26], or it is $\mathrm{r}_p = (0.883 \pm 0.014) \cdot 10^{-13}$ cm according to [27]. The Review of Particle Physics gives for the charge radius of the proton the value $\mathrm{r}_p = (0.875 \pm 0.007) \cdot 10^{-13}$ cm. With $a = 10^{-16}$ cm it follows from Eq.(16) that the number of all lattice points in the cubic lattice is

$$\mathrm{N} = 2.854 \cdot 10^9 \cong 1\,418^{\,3}\,. \tag{17}$$

When the number of the grid points of a cubic lattice is derived from the volume of a sphere one cannot arrive at an integer number to the third. The radius of the $\pi^\pm$ mesons has also been measured [28] and after further analysis [29] was found to be $0.83 \cdot 10^{-13}$ cm. A much earlier measurement [30] found r_π at $(0.86 \pm 0.14) \cdot 10^{-13}$ cm. The Review of Particle Physics does not give a value for r_π. Within the uncertainty of the radii we have $\mathrm{r}_p = \mathrm{r}_\pi$. And according to [31] the charge radius of Σ^- is $(0.78 \pm 0.10) \cdot 10^{-13}$ cm.

If the oscillations are parallel to an axis, the limitation of the group velocity is taken into account, that means if Eq.(14) applies and the absolute values of the frequencies are taken, then the value of the integral in Eq.(15) is

π^2. With N $= 2.854 \cdot 10^9$ and $\nu_0 = \mathrm{c}/2\pi a$ follows from Eq.(15) that the sum of the energy of the frequencies of the basic mode corrected for the group velocity limitation is $\mathrm{E}_{corr} = 1.418 \cdot 10^9\,$erg. That means that the energy is $6.56 \cdot 10^{12}$ times larger than $\mathrm{E}(\mathrm{m}(\pi^0))$. This discrepancy is inevitable, because the basic frequency of the Fourier spectrum after a collision on the order of 10^{-23} sec duration is $\nu = 10^{23}$ sec^{-1}, which means, when $\mathrm{E} = \mathrm{h}\nu$, that one basic frequency alone contains an energy of about $9\,\mathrm{m}(\pi^0)\mathrm{c}^2$.

To eliminate this discrepancy we use, instead of the simple form $\mathrm{E} = \mathrm{h}\nu$, the complete quantum mechanical energy of a linear oscillator as given by Planck

$$E = \frac{h\nu}{e^{h\nu/kT} - 1} \ . \tag{18}$$

This equation was already used by B&K for the determination of the specific heat of cubic crystals or solids. Equation (18) calls into question the value of the temperature T in the interior of a particle. We determine T empirically with the formula for the internal energy of solids

$$u = \frac{R\Theta}{e^{\Theta/T} - 1} \ , \tag{19}$$

which is from Sommerfeld [32]. In this equation $\mathrm{R} = \mathrm{N} \cdot \mathrm{k} = 2.854 \cdot 10^9\,\mathrm{k}$, where k is Boltzmann's constant, and Θ is the characteristic temperature introduced by Debye [33] for the explanation of the specific heat of solids. It is $\Theta = h\nu_m/\mathrm{k}$, where ν_m is a maximal frequency. In the case of the oscillations making up the π^0 meson the maximal frequency is $\nu_m = \pi\nu_0$, therefore $\nu_m = 1.5 \cdot 10^{26}$ sec^{-1}, and we find that $\Theta = 7.2 \cdot 10^{15}\,$K.

In order to determine T we set the internal energy u equal to $\mathrm{m}(\pi^0)\mathrm{c}^2$. It then follows from Eq.(19) that $\Theta/\mathrm{T} = 30.20$, or $\mathrm{T} = 2.38 \cdot 10^{14}\,$K. That means that Planck's formula (18) changes Eq.(15) into

$$\mathrm{E}_\nu(\pi^\pm) = \frac{\mathrm{Nh}\nu_0}{2\pi(\mathrm{e}^{\mathrm{h}\nu/\mathrm{kT}} - 1)} \int\limits_{-\pi}^{\pi} \phi\, d\phi \, , \tag{20}$$

This introduces a factor

$$1/(\mathrm{e}^{\Theta/T} - 1\,) \cong 1/\mathrm{e}^{30.2} = 1/(1.305 \cdot 10^{13})\, , \tag{21}$$

into Eq.(15). In other words, if we determine the temperature T of the particle through Eq.(19), and correct Eq.(15) accordingly then we arrive at a sum of the oscillation energies in the π^0 meson which is

$$\sum_{1}^{N} \mathrm{E}_\nu = 1.0866 \cdot 10^{-4}\,\mathrm{erg} = 67.82\,\mathrm{MeV}\,, \tag{22}$$

whereas $\mathrm{m}(\pi^0)\mathrm{c}^2/2 = 67.488\,\mathrm{MeV}$. The sum of the energies of N one-dimensional photon oscillations in the π^0 meson lattice is $0.502\,\mathrm{E}(\mathrm{m}(\pi^0))$. We have to double this amount because standing waves consist of two waves traveling in opposite direction with the same absolute value of the frequency. The sum of the energy of the oscillations in the π^0 meson is therefore

$$\mathrm{E}_\nu(\pi^0)(theor) = 2.1732 \cdot 10^{-4}\,\mathrm{erg} = 135.64\,\mathrm{MeV} = 1.005\,\mathrm{E}(\mathrm{m}(\pi^0))(exp)\,, \tag{23}$$

if the oscillations are parallel to the ϕ axis. The energy in the measured mass of the π^0 meson and the energy in the sum of the oscillations agree fairly well, considering the uncertainties of the parameters involved. The theoretical mass of the η meson is then $\mathrm{m}(\eta)(theor) = 4 \cdot \mathrm{m}(\pi^0) = 542.56\,\mathrm{MeV} = 0.991\,\mathrm{m}(\eta)(exp)$, and the theoretical mass of the Λ baryon, the superposition of two η mesons, is then $\mathrm{m}(\Lambda)(theor) = 8 \cdot \mathrm{m}(\pi^0) = 1085.1\,\mathrm{MeV} = 0.9726\,\mathrm{m}(\Lambda)(exp)$.

To sum up: The π^0 meson is formed when a γ-ray collides with a proton, $\gamma + \mathrm{p} \to \pi^0 + \mathrm{p}$. By the collision the incoming γ-ray is converted into a packet of standing electromagnetic waves, the π^0 meson. After 10^{-16} seconds the wave packet decays into two electromagnetic waves, $\pi^0 \to \gamma\gamma$. Electromagnetic waves prevail throughout the entire process. The energy in the rest mass of the π^0 meson and the other particles of the γ-branch is correctly given by the sum of the energy of standing electromagnetic waves in a cube, if the energy of the oscillations is determined by Planck's formula for the energy of a linear oscillator. *The π^0 meson is like an adiabatic, cubic black body filled with standing electromagnetic waves.* A black body of a given size and temperature can certainly contain the energy in the rest mass of the π^0 meson, which is $\mathrm{O}(10^{-4})$ erg, if only the frequencies are sufficiently high. We know from Bose's work [34] that Planck's formula applies to a photon gas as well. For all γ-branch particles we have found a simple mode of standing electromagnetic waves. Since the equation determining the frequency of the standing waves is quadratic it follows *automatically* that for each positive frequency there is also a negative frequency of the same absolute value, that means that for

each particle there exists also an antiparticle. For the explanation of the stable mesons and baryons of the γ-branch *we use only photons, nothing else.* This is a rather conservative explanation of the π^0 meson and the γ-branch particles. *We do not use hypothetical particles.*

From the frequency distributions of the standing waves follow the ratios of the masses of the particles which obey the integer multiple rule. It is important to note that in this theory the ratios of the masses of the γ-branch particles to the mass of the π^0 meson *do not depend* on the sidelength of the lattice, and the distance between the lattice points, neither do they depend on the strength of the force between the lattice points nor on the mass of the lattice points. The mass ratios are determined only by the spectra of the frequencies of the standing electromagnetic waves.

5 The neutrino branch particles

The masses of the neutrino branch, the $\pi^\pm$, $\mathrm{K}^{\pm,0}$, n, $\mathrm{D}^{\pm,0}$ and $\mathrm{D}_s^\pm$ particles, are integer multiples of the mass of the $\pi^\pm$ mesons times a factor 0.85 ± 0.02 as we stated before. We assume, as appears to be quite natural, that the $\pi^\pm$ mesons and the other particles of the neutrino branch *consist of the same particles into which they decay*, that means in the case of the $\pi^\pm$ mesons of muon neutrinos ν_μ, anti-muon neutrinos $\bar{\nu}_\mu$, electron neutrinos ν_e, anti-electron neutrinos $\bar{\nu}_e$ and of an electron or positron, as exemplified by the decay sequence $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$, $\mu^\pm \rightarrow \mathrm{e}^\pm + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e)$. The absence of an electron neutrino ν_e in the decay branches of π^- or of an anti-electron neutrino $\bar{\nu}_e$ in the decay branches of π^+ can be explained with the composition of the electron or positron, which will be discussed in Section 11. The existence of neutrinos and antineutrinos is unquestionable. Since the particles of the ν-branch decay through weak decays, we assume, as appears likewise to be natural, that *the weak nuclear force holds the particles of the ν-branch together.* This assumption has far reaching consequences, it is not only fundamental for the explanation of the $\pi^\pm$ mesons, but leads also to the explanation of the $\mu^\pm$ mesons and ultimately to the explanation of the mass of the electron. The existence of the weak nuclear force is unquestionable. Since the range of the weak interaction, which is about 10^{-16} cm [22], is only about a thousandth of the diameter of the particles, which is about 10^{-13} cm, the weak force can hold particles together only if the particles have a lattice structure, just as macroscopic crystals are held together by microscopic forces

between atoms. In the absence of a central force which originates in the center of the particle and extends throughout the entire particle, as the Coulomb force, the configuration of the particle is not spherical but cubic, reflecting the very short range of the weak nuclear force. We will show that the energy in the rest mass of the ν-branch particles is the energy in the oscillations of a cubic lattice consisting of electron neutrinos and muon neutrinos and their antiparticles, plus the energy in the rest masses of the neutrinos, plus a small part with the energy in the electric charges the particle carries.

First it will be necessary to outline the basic aspects of diatomic lattice oscillations. In *diatomic* lattices the lattice points have alternately the masses m and M, as with the masses of the electron neutrinos m(ν_e) and muon neutrinos m(ν_μ). The classic example of a diatomic lattice is the salt crystal with the masses of the Na and Cl atoms in the lattice points. The theory of diatomic harmonic lattice oscillations was started by Born and v. Karman [14]. They first discussed a diatomic chain. The equation of motions in the chain are according to Eq.(22) of B&K

$$\mathrm{m}\ddot{u}_{2n} = \alpha(u_{2n+1} + u_{2n-1} - 2u_{2n})\,, \tag{24}$$

$$\mathrm{M}\ddot{u}_{2n+1} = \alpha(u_{2n+2} + u_{2n} - 2u_{2n+1})\,, \tag{25}$$

where the u_n are the displacements, n an integer number and α a constant characterizing the force between the particles. Eqs.(24,25) are solved with

$$u_{2n} = Ae^{i(\omega\, t\, +\, 2n\phi)}, \tag{26}$$

$$u_{2n+1} = Be^{i(\omega\, t\, +\, (2n+1)\phi)}\,, \tag{27}$$

where A and B are constants and ϕ is given by $\phi = 2\pi a/\lambda$ as in Eq.(6). a is the lattice constant as before and λ the wavelength, $\lambda = \mathrm{n}a$. The solutions of Eqs.(26,27) are obviously periodic in time and space and describe again standing waves. Using (26,27) to solve (24,25) leads to a secular equation from which according to Eq.(24) of B&K the frequencies of the oscillations of the chain follow from

$$4\pi^2\nu_\pm^2 = \alpha/\mathrm{Mm}\cdot((\mathrm{M}+\mathrm{m}) \pm \sqrt{(\mathrm{M}-\mathrm{m})^2 + 4\mathrm{mM}\cos^2\phi}\,)\,. \tag{28}$$

Longitudinal and transverse waves are distinguished by the minus or plus sign in front of the square root in (28).

6 The masses of the ν-branch particles

The characteristic case of the neutrino branch particles are the $\pi^{\pm}$ mesons which can be created in the process γ + p $\rightarrow$ π^- + π^+ + p. A photon impinges on a proton and is converted in 10^{-23} sec into a pair of particles of opposite charge. A simple example of the creation of a ν-branch particle by strong interaction is the case p + p $\rightarrow$ p + p + π^- + π^+. Fourier analysis dictates that a continuum of frequencies must be in the collision products. The waves must be standing waves in order to be part of the rest mass of a particle. The $\pi^{\pm}$ mesons decay via $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ (99.98770%) followed by $\mu^{\pm} \rightarrow \mathrm{e}^{\pm} + \bar{\nu}_{\mu}(\nu_{\mu}) + \nu_e(\bar{\nu}_e)$ ($\approx$ 100%). Only $\mu^{\pm}$ mesons, which decay into $\mathrm{e}^{\pm}$ and neutrinos, and neutrinos result from the decay of the $\pi^{\pm}$ mesons. If the particles consist of the particles into which they decay, then the $\pi^{\pm}$ mesons are made of neutrinos, antineutrinos and $\mathrm{e}^{\pm}$. Since neutrinos interact through the weak force which has a range of O(10^{-16}) cm according to p.25 of [22], and since the size of the $\pi^{\pm}$ mesons [30] is on the order of 10^{-13} cm, *the ν-branch particles must have a lattice with N neutrinos*, N being the same as in Eq.(17). It is not known with certainty that neutrinos actually have a rest mass as was originally suggested by Bethe [35] and Bahcall [36] and what the values of m(ν_e) and m(ν_{μ}) are. However, the results of the Super-Kamiokande [37] and the Sudbury [38] experiments indicate that the neutrinos have rest masses. Different rest masses of the electron neutrino, muon neutrino and tau neutrino guarantee that the three neutrino types are different. And, as the experiments show, they are different. Otherwise the neutrinos of the three types do not differ, they do not have charge and have the same spin.

The neutrino lattice must be diatomic, meaning that the lattice points have alternately larger (m(ν_{μ})) and smaller (m(ν_e)) masses. We will retain the traditional term diatomic. *The term neutrino lattice will refer to a lattice consisting of neutrinos and antineutrinos.* The lattice we consider is shown in Fig. 2. Since the neutrinos have spin 1/2 this is a four-Fermion lattice which is required for the explanation of the weak decays. The first investigation of cubic Fermion lattices in context with the elementary particles was made by Wilson [13]. A neutrino lattice is electrically neutral. Since we do not know the interaction of the electron with a neutrino lattice we cannot consider lattices with a charge.

The neutrino lattice oscillations take care of the continuum of frequencies which must, according to Fourier analysis, be present after the high energy collision which created the particle. We will, for the sake of simplicity, first

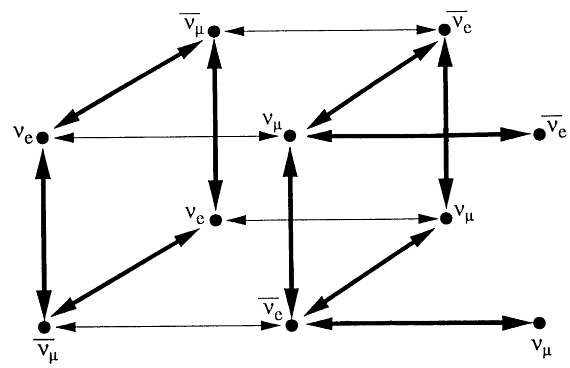

Fig. 2: A cell in the neutral neutrino lattice of the $\pi^{\pm}$ mesons. Bold lines mark the forces between neutrinos and antineutrinos. Thin lines mark the forces between either neutrinos only, or antineutrinos only.

set the sidelength of the lattice at 10^{-13} cm that means approximately equal to the size of the nucleon. The lattice then contains about 10^9 lattice points. The sidelength of the lattice does not enter Eq.(28) for the frequencies of diatomic oscillations. The calculation of the ratios of the masses is consequently independent of the size of the lattice, as was the case with the γ-branch. The size of the lattice can be explained with the pressure which the lattice oscillations exert on a crossection of the lattice. The pressure cannot exceed Young's modulus of the lattice. We require that the lattice is isotropic.

From the frequency distribution of the axial diatomic oscillations (Eq.28), shown in Fig. 3, follows the group velocity $\mathrm{d}\omega/\mathrm{dk} = 2\pi a\, d\nu/d\phi$ at each point ϕ. With $\nu = \nu_0 f(\phi)$ and $\nu_0 = \sqrt{\alpha/4\pi^2\mathrm{M}} = \mathrm{c}/2\pi a$ as in Eq.(10) we find

$$\mathrm{c}_g = d\omega/dk = a\sqrt{\alpha/\mathrm{M}} \cdot df(\phi)/d\phi\,. \tag{29}$$

In order to determine the value of $\mathrm{d}\omega/dk$ we have to know the value of $\sqrt{\alpha/\mathrm{M}}$. From Eq.(9) for α follows that $\alpha = a\,c_{44}$ in the isotropic case with central forces. The group velocity is therefore

$$\mathrm{c}_g = \sqrt{a^3 c_{44}/\mathrm{M}} \cdot df/d\phi\,. \tag{30}$$

In Eq.(29) we now set $a\sqrt{\alpha/\mathrm{M}} = \mathrm{c}$, as in Eq.(11), where c is the velocity of

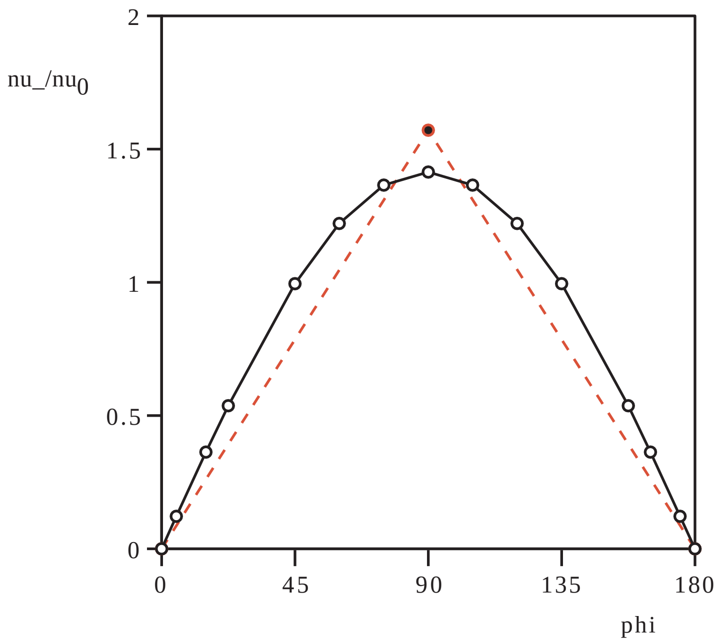

Fig. 3: The frequency distribution ν_{-}/ν_0 of the basic diatomic mode according to Eq.(24) with M/m = 100. The dashed line shows the distribution of the frequencies corrected for the group velocity limitation.

light. It follows that

$$c_g = c \cdot df/d\phi\,, \tag{31}$$

as it was with the γ-branch, only that now on account of the rest masses of the neutrinos the group velocity must be smaller than c, so the value of df/dϕ is limited to <1, but $c_g \cong$ c, which is a necessity because the neutrinos in the lattice soon approach the velocity of light as we will see. Equation (31) applies regardless whether we consider ν_+ or ν_- in Eq.(28). That means that there are no separate transverse oscillations with their theoretically higher frequencies.

The rest mass M of the heavy neutrino can be determined with lattice theory from Eq.(30) as we have shown in [12]. This involves the inaccurately known compression modulus of the proton. We will, therefore, rather determine the rest mass of the muon neutrino with Eq.(33), which leads to m(ν_μ)

$\approx 50\,\text{milli-eV/c}^2$. It can be verified easily that $m(\nu_\mu) = 50\,\text{milli-eV/c}^2$ makes sense. The energy of the rest mass of the $\pi^\pm$ mesons is 139 MeV, and we have $N/4 = 0.7135{\cdot}10^9$ muon neutrinos and the same number of anti-muon neutrinos with an energy of about 50 milli-eV. It follows that the energy in the rest masses of all muon and anti-muon neutrinos in $\pi^\pm$ is 71.35 MeV, that is 51% of the energy of the rest mass of the $\pi^\pm$ mesons, $m(\pi^\pm)c^2 = 139.57\,\text{MeV}$. A very small part of $m(\pi^\pm)c^2$ goes, as we will see, into the electron neutrino masses, the rest of the energy in $\pi^\pm$ is in the lattice oscillations.

The energy in the rest mass of the $\pi^\pm$ mesons is the sum of the oscillation energies plus the sum of the energy in the rest masses of the neutrinos, plus the energy in $e^\pm$.

The $\pi^\pm$ mesons are like cubic black bodies filled with oscillating neutrinos. For the sum of the energies of the frequencies we use Eq.(20), with the same N and ν_0 we used for the γ-branch. For the integral in Eq.(20) of the axial diatomic frequencies corrected for the group velocity limitation we find the value $\pi^2/2$ as can be easily derived from the plot of the corrected frequencies in Fig. 3. The value of the integral in Eq.(20) for the axial diatomic frequencies $\nu = \nu_0\phi$ is 1/2 of the value π^2 of the same integral in the case of axial monatomic frequencies, because in the latter case the increase of the corrected frequencies continues to $\phi = \pi$, whereas in the diatomic case the increase of the corrected frequencies ends at $\pi/2$, see Fig. 3. We consider c_g to be so close to c that it does not change the value of the integral in Eq.(20) significantly. It can be calculated that the time average of the velocity of the electron neutrinos in the $\pi^\pm$ mesons is $\bar{v} = 0.99989\,c$ if $m(\nu_e) = 0.365\,\text{milli-eV/c}^2$ as will be shown in Eq.(67). Consequently we find that the sum of the energies of the corrected diatomic neutrino frequencies is $0.5433{\cdot}10^{-4}$ erg $= 33.91\,\text{MeV}$. We double this amount because we deal with standing waves or the superposition of two waves of the same energy and find with Eq.(20) that the energy of the neutrino oscillations in $\pi^\pm$ is

$$E_\nu(\pi^\pm) \cong 1/2 \cdot E_\nu(\pi^0) = 67.82\,\text{MeV} = 0.486\,m(\pi^\pm)c^2\,, \tag{32}$$

or that $\approx$ *1/2 of the energy of $\pi^\pm$ is in the oscillation energy $E_\nu(\pi^\pm)$.*

In order to determine the sum of the rest masses of the neutrinos we make use of $E_\nu(\pi^\pm)$ and obtain an approximate value of the sum of the rest masses of the neutrinos in $\pi^\pm$ from

$$m(\pi^\pm)c^2 - E_\nu(\pi^\pm) = \sum\,[m(\nu_\mu) + m(\bar{\nu}_\mu) + m(\nu_e) + m(\bar{\nu}_e)]c^2 = 71.75\,\text{MeV}\,, \tag{33}$$

and find that $\approx$ *1/2 of the energy of $\pi^{\pm}$ is in the neutrino rest masses.* Since nothing else but the electric charge contributes to the rest mass of $\pi^{\pm}$ it appears that in a good approximation the oscillation energy in $\pi^{\pm}$ is equal to the energy in the sum of the neutrino rest masses in $\pi^{\pm}$, i.e.

$$\mathrm{E}_{\nu}(\pi^{\pm}) \cong \Sigma\,\mathrm{m(neutrinos)}(\pi^{\pm})\mathrm{c}^2 = \mathrm{N}/2 \cdot (\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_e))\mathrm{c}^2 \cong 1/2 \cdot \mathrm{m}(\pi^{\pm})\mathrm{c}^2. \tag{34}$$

This applies only to the neutral neutrino lattice of the pion, the consequences of the charge of $\pi^{\pm}$ have not been considered.

If $\mathrm{m}(\nu_e) \ll \mathrm{m}(\nu_\mu)$ and $\mathrm{m}(\nu_\mu) = \mathrm{m}(\bar{\nu}_\mu)$, as we will justify later, we arrive with Eq.(33) and $\mathrm{N}/2 = 1.427{\cdot}10^9$ at an approximate value for the rest mass of the muon neutrino

$$\mathrm{m}(\nu_\mu) \approx 50\,\text{milli-eV/c}^2.$$

An accurate value of $\mathrm{m}(\nu_\mu)$ will be given later, Eq.(70).

The sum of the energy of the rest masses of all neutrinos in $\pi^{\pm}$, Eq.(33), plus the oscillation energy, Eq.(32), neglecting the energy in $\mathrm{e}^{\pm}$, gives the theoretical rest mass of the $\pi^{\pm}$ mesons which is, since we used $\mathrm{m}(\pi^{\pm})$ in the determination of the neutrino rest masses with Eq.(33), equal to the experimental rest mass of 139.57 MeV/c^2.

A cubic lattice and conservation of neutrino numbers during the reaction $\gamma + \mathrm{p} \rightarrow \pi^+ + \pi^- + \mathrm{p}$ *necessitates* that the π^+ and π^- lattices contain just as many electron neutrinos as anti-electron neutrinos. If the lattice is cubic it must have a center neutrino (Fig. 4). Conservation of neutrino numbers requires furthermore that the center neutrino of π^+ is matched by an antineutrino in π^-. In the decay sequence of (say) the π^- meson $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ and $\mu^- \rightarrow \mathrm{e}^- + \nu_\mu + \bar{\nu}_e$ an electron neutrino ν_e does not appear. But since (N - 1)/4 electron neutrinos ν_e must be in the π^- lattice it follows that (N - 1)/4 electron neutrinos must go with the electron emitted in the μ^- decay.

We must now be more specific about N, which is an odd number, because a cubic lattice (Fig. 4) has a center particle, just as the NaCl lattice. In the $\pi^{\pm}$ mesons there are then (N - 1)/4 muon neutrinos ν_μ and the same number of anti-muon neutrinos $\bar{\nu}_\mu$, as well as (N - 1)/4 electron neutrinos ν_e and the same number of anti-electron neutrinos $\bar{\nu}_e$, *plus* a center neutrino or antineutrino. We replace N - 1 by N′. Since N′ differs from N by only one in 10^9 we have N′ $\cong$ N. Although the numerical difference between N and N′ is negligible we cannot consider any integer number N because that would mean that there could be fractions of a neutrino. N′ is an even number, because

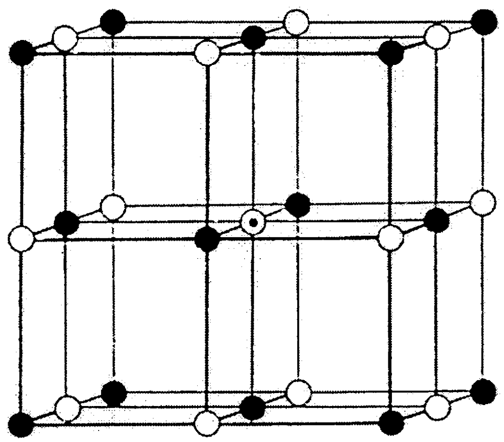

Fig. 4: The center of a NaCl lattice. (After Born and Huang).

each cell (Fig. 2) of the lattice (Fig. 4) consists of four pairs of neutrinos.

The *antiparticle* of the π^+ meson is the particle in which all frequencies of the neutrino lattice oscillations have been replaced by frequencies with the opposite sign, all neutrinos replaced by their antiparticles and the positive charge replaced by the negative charge. If, as we will show, the antineutrinos have the same rest mass as the neutrinos it follows that the antiparticle of the π^+ meson has the same mass as π^+ but opposite charge, i.e. is the π^- meson. As we will see, the explanation of the mass of the $\pi^\pm$ mesons opens the door to the explanation of the mass of the muon and of the electron.

Now we turn to the K mesons whose mass is $\mathrm{m}(\mathrm{K}^\pm) = 0.8843 \cdot 4\mathrm{m}(\pi^\pm)$. The primary decay of the $\mathrm{K}^\pm$ mesons $\mathrm{K}^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ (63.5%) leads to the same end products as the $\pi^\pm$ meson decay $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ (99.98%). From this and the decay of the $\mu^\pm$ mesons we learn that the K mesons must, at least partially, be made of the same four neutrino types as in the $\pi^\pm$ mesons namely of muon neutrinos, anti-muon neutrinos, electron neutrinos and anti-electron neutrinos and their oscillation energies. However the $\mathrm{K}^\pm$ mesons cannot be solely the second mode of the lattice oscillations of the $\pi^\pm$ mesons, because the second mode of the $\pi^\pm$ mesons has an energy of

$$\begin{aligned} \mathrm{E}((2.)\pi^\pm) &= 4\mathrm{E}_\nu(\pi^\pm) + \mathrm{N}/2 \cdot (\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_\mathrm{e}))\,\mathrm{c}^2 \\ &\cong 2\mathrm{m}(\pi^\pm)\mathrm{c}^2 + 1/2 \cdot \mathrm{m}(\pi^\pm)\mathrm{c}^2 = 348.92\,\mathrm{MeV}\,, \end{aligned} \tag{35}$$

with $2\mathrm{E}_\nu(\pi^\pm) \cong \mathrm{m}(\pi^\pm)\mathrm{c}^2$ and $\mathrm{N}/2 \cdot (\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_e)) \cong \mathrm{m}(\pi^\pm)/2$ from Eqs.

(33,34). The 348.9 MeV characterize the second or (2.) mode of the $\pi^\pm$ mesons, which fails $m(K^\pm)c^2 = 493.7$ MeV by a wide margin.

The concept that the $K^\pm$ mesons are alone a higher mode of the $\pi^\pm$ mesons also contradicts our point that the particles consist of the particles into which they decay. The decays $K^\pm \rightarrow \pi^\pm + \pi^0$ (21.13%), as well as $K^+ \rightarrow \pi^0 + e^+ + \nu_e$ (4.87%), called K^+_{e3}, and $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$ (3.27%), called $K^+_{\mu 3}$, make up 29.27% of the $K^\pm$ meson decays. A π^0 meson figures in each of these decays. If we add the energy in the rest mass of a π^0 meson $m(\pi^0)c^2 = 134.97$ MeV to the 348.9 MeV in the second mode of the $\pi^\pm$ mesons then we arrive at an energy of 483.9 MeV, which is 98.0% of $m(K^\pm)c^2$. Therefore we conclude that the $K^\pm$ mesons consist of the second mode of the $\pi^\pm$ mesons *plus* a π^0 meson or are the state $(2.)\pi^\pm + \pi^0$. Then it is natural that π^0 mesons from the π^0 component in the $K^\pm$ mesons are among the decay products of the $K^\pm$ mesons.

The average factor 0.85 ± 0.025 which appears in Eq.(3) for the ratios of the masses of the particles of the ν-branch to the mass of the $\pi^\pm$ mesons is a consequence of the neutrino rest masses. They make it impossible that the ratios of the particle masses are outright integer multiples because the particles consist of the energy in the neutrino oscillations and in the neutrino rest masses which are independent of the order of the lattice oscillations. Since the contribution in percent of the neutrino rest masses to the ν-branch particle masses decreases with increased particle mass the factor in front of the mass ratios of the ν-branch particles must decrease with increased particle mass.

The $K^0,\overline{K^0}$ mesons have a rest mass $m(K^0,\overline{K^0}) = 1.00809\, m(K^\pm)$, or it is $m(K^0,\overline{K^0}) = 0.99984\,(m(K^\pm) + \alpha_f \cdot 4m(\pi^\mp))$. We obtain the K^0 meson if we superpose onto the second mode of the $\pi^\pm$ mesons instead of a π^0 meson a basic mode of the $\pi^\pm$ mesons with a charge opposite to the charge of the second mode of the $\pi^\pm$ meson. The K^0 and $\overline{K^0}$ mesons, or the state $(2.)\pi^\pm + \pi^\mp$, is made of neutrinos and antineutrinos only, without a photon component, because the second mode of $\pi^\pm$ as well as the basic mode $\pi^\mp$ consist of neutrinos and antineutrinos only. The K^0 meson has a measured mean square charge radius $\langle r^2 \rangle = -0.077 \pm 0.010\,\mathrm{fm}^2$ according to [39], which can only be if there are two charges of opposite sign within K^0, as this model implies. Since the mass of a $\pi^\pm$ meson is by $4.59\,\mathrm{MeV/c^2}$ larger than the mass of a π^0 meson the mass of K^0 should be larger than $m(K^\pm)$, and indeed $m(K^0) - m(K^\pm) = 3.972\,\mathrm{MeV/c^2}$ according to [2]. Similar differences occur with $m(D^\pm) - m(D^0)$ and $m(\Xi^0_c) - m(\Xi^+_c)$. The decay $K^0_S \rightarrow \pi^+ + \pi^-$ (68.6%)

creates directly the π^+ and π^- mesons which are part of the $(2.)\pi^\pm + \pi^\mp$ structure of K^0 we have suggested. The decay $K^0_S \rightarrow \pi^0 + \pi^0$ (31.4%) apparently originates from the 2γ branch of electron positron annihilation. Both decays account for 100% of the decays of K^0_S. The decay $K^0_L \rightarrow 3\pi^0$ (21.1%) apparently comes from the 3γ branch of electron positron annihilation. The two decays of K^0_L called $K^0_{\mu 3}$ into $\pi^\pm\,\mu^\mp\,\nu_\mu$ (27.18%) and K^0_{e3} into $\pi^\pm\,e^\mp\,\nu_e$ (38.79%) which together make up 65.95% of the K^0_L decays apparently originate from the decay of the second mode of the $\pi^\pm$ mesons in the K^0 structure, either into $\mu^\mp + \nu_\mu$ or into $e^\mp + \nu_e$. The same types of decay, apparently tied to the $(2.)\pi^\pm$ mode, accompany also the $K^\pm$ decays $K^\pm \rightarrow \pi^\pm\pi^0$ (20.92%) in which, however, a π^0 meson replaces the $\pi^\pm$ mesons in the K^0_L decay products. Our rule that the particles consist of the particles into which they decay also holds for the K^0 and $\overline{K^0}$ mesons. The explanation of the K^0,$\overline{K^0}$ mesons with the state $(2.)\pi^\pm + \pi^\mp$ confirms that the state $(2.)\pi^\pm + \pi^0$ was the correct choice for the explanation of the $K^\pm$ mesons. The state $(2.)\pi^\pm + \pi^\mp$ is also crucial for the explanation of the absence of spin of the K^0,$\overline{K^0}$ mesons, as we will see later.

The neutron whose mass is $m(n) = 0.95156 \cdot 2m(K^\pm)$ is either the superposition of a K^+ and a K^- meson or of a K^0 meson and a $\overline{K^0}$ meson. As has been shown in [67], the spin rules out a neutron consisting of a K^+ and a K^- meson. On the other hand, the neutron can be the superposition of a K^0 and a $\overline{K^0}$ meson which guarantees that the neutron consists of neutrinos without a photon component. In this case the neutron lattice contains at each lattice point a $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrino quadrupole because in each K^0 and $\overline{K^0}$ meson are neutrino pairs at the lattice points, and there is a single quadrupole of positive and negative electric charges because each K^0 and $\overline{K^0}$ meson carries a pair of opposite elementary electric charges. There must be opposite charges in the neutron because it has a mean square charge radius $\langle r^2\rangle = -\,0.1161\,\mathrm{fm}^2$ [2]. The lattice oscillations in the neutron must be a coupled pair in order for the neutron to have spin 1/2, just as the Λ baryon with spin 1/2 is a superposition of two η mesons. With $m(K^0)(\mathit{theor}) = m(K^\pm) + 4\,\mathrm{MeV/c^2} = 487.9\,\mathrm{MeV/c^2}$ from above it follows that $m(n)(\mathit{theor}) \approx 2m(K^0)(\mathit{theor}) \approx 975.8\,\mathrm{MeV/c^2} = 1.04\,m(n)(\mathit{exp})$.

The proton, whose mass is $m(p) = 0.99862\,m(n)$, does not decay and does not tell which particles it is made of. However, we learn about the structure of the proton through the decay of the neutron $n \rightarrow p + e^- + \bar{\nu}_e$ (100%). An electron and one single anti-electron neutrino is emitted when the neutron decays and 1.29333 MeV are released. But there is no place for a permanent

vacancy of a single missing neutrino and for a small amount of permanently missing oscillation energy in a nuclear lattice. As it appears all anti-electron neutrinos are removed from the structure of the neutron in the neutron decay and converted into the kinetic energy of the decay products. This type of process will be explained in Section 9. On the other hand, it is certain that the proton consists of a neutrino lattice because the neutron has a neutrino lattice. The proton carries a net positive elementary electric charge because the neutron carries an $e^+e^-e^+e^-$ quadrupole, of which one e^- is lost in the β-decay. The concept that the proton carries just one elementary electric charge has been abandoned a long time ago when it was said that the proton consists of three quarks carrying fractional electric charges. Each elementary charge in the proton has a magnetic moment, all of them point in the same direction because the spin of the one e^- must be opposite to the spin of the two e^+. Each magnetic moment of the elementary charges has a g-factor $\cong 2$. All three electric charges in the proton must then have a g-factor ≈ 6, whereas the measured g-factor of the magnetic moment of the proton is g(p) $= 5.585 = 0.93 \cdot 6$.

The $D^\pm$ mesons with $m(D^\pm) = 0.9954\,(m(p) + m(\bar{n}))$ are the superposition of a proton and an antineutron of opposite spin or of their antiparticles, whereas the superposition of a proton and a neutron with the same spin creates the deuteron with spin 1 and a mass $m(d) = 0.9988\,(m(p) + m(n))$. In this case the proton and neutron interact with the strong force, nevertheless the deuteron consists of a neutrino lattice with standing waves. The $D_s^\pm$ mesons seem to be made of a body centered cubic lattice (Fig. 5) as described in [40].

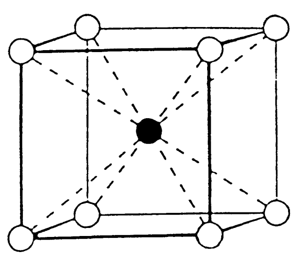

Fig. 5: A body-centered cell. (After Born and Huang). In the center of a $D_s^\pm$ cell is a τ neutrino, in the corners are $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos, as in Fig. 2.

Summing up: The particles of the ν-branch consist of oscillating neutrinos and one or more positive and/or negative elementary electric charges. The characteristic feature of the ν-branch particles is the cubic lattice consisting of $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos. The rest mass of the ν-branch particles is the sum of the rest masses of the neutrinos and antineutrinos in the lattice plus the mass in the energy of the lattice oscillations plus the mass in the electric charges. The existence of the neutrino lattice is a necessity if one wants to explain the spin, or the absence of spin, of the ν-branch particles. *We do not use hypothetical particles* for the explanation of the ν-branch particles, just as we did not use hypothetical particles for the explanation of the γ-branch particles.

7 The weak force in the interior of the particles

After we have explained the masses of the stable mesons and baryons with cubic lattices consisting of either photons or of neutrinos, we can now determine the strength of the weak and the strength of the strong nuclear forces. Both are 70 years old puzzles. We will use lattice theory to determine the strength of the weak force which holds the lattices of the elementary particles together. We will then show that the strong force between two elementary particles is nothing but the sum of the unsaturated weak forces emanating from the lattice points at the surface of the lattice.

In order to determine the force in the interior of the cubic lattices with which we have explained the particle masses we will, as we have done before in [23], use a classical paper by Born and Landé [41], (B&L), dealing with the potential and compressibility of regular ionic crystals. It is essential to realize that,

- *for the existence of a cubic lattice it is necessary that the force between the lattice points has an attractive part and a repulsive part.*

- Otherwise the lattice would not be stable and collapse.

For the ionic crystals considered by B&L the Coulomb force between the ions is the attractive force, whereas the repulsive force originates from the electron clouds surrounding the ions. When the electron clouds of the ions

approach each other during the lattice oscillations they repel each other. The magnitude of the repulsive force is not known per se and has to be determined from the properties of the crystal.

We will follow exactly the procedure in B&L in order to see whether their theory is also applicable to a cubic lattice made of neutrinos. In this case the Coulomb force is, of course, irrelevant. As B&L do, we say that the potential of a cell of the neutrino lattice has an attractive part $-\,\mathrm{a}/\delta$ and a repulsive part $+\,\mathrm{b}/\delta^{\mathrm{n}}$ with the unknown exponent n. δ is the distance in the direction between two neutrinos of the same type, either muon neutrinos and anti-muon neutrinos or electron neutrinos and anti-electron neutrinos.

The potential of a cell in an ionic cubic lattice is of the form

$$\phi = -\,\mathrm{a}/\delta + \mathrm{b}/\delta^{\mathrm{n}}\,, \tag{36}$$

Eq.(1) of B&L. The constant b is eliminated with the equilibrium condition $\mathrm{d}\phi/\mathrm{d}\delta = 0$. Consequently

$$\mathrm{b} = \frac{\mathrm{a}}{\mathrm{n}}\,\delta_0^{\mathrm{n}-1}\,, \tag{37}$$

where δ_0 is the lattice constant. The unknown exponent n of δ in Eq.(36) was determined by B&L with the help of the compression modulus κ which is known for ionic crystals. κ is defined by

$$\kappa = -\,1/V \cdot dV/dp\,. \tag{38}$$

The compression modulus of an ionic lattice is given by

$$\kappa = 9\,\delta_0^4/\mathrm{a}\,(\mathrm{n}-1)\,, \tag{39}$$

Eq.(4) in B&L. The interaction constant a of the Coulomb force in cubic ionic crystals resulting from the contributions of all ions on a single ion is given by Eq.(5) of B&L, it is

$$\mathrm{a} = 13.94\,\mathrm{e}^2 = 3.2161 \cdot 10^{-18}\,\mathrm{erg}\cdot\mathrm{cm}\,, \tag{40}$$

where e is the elementary electric charge. This equation is fundamental for the theory of ionic lattices and is based on an earlier paper by Madelung [42]. Consequently we find that

$$(\mathrm{n}-1) = 10.33\,\mathrm{r}_0^4/\mathrm{e}^2\kappa\,, \tag{41}$$

where $r_0 = \delta_0/2$ is the distance between a pair of neighboring Na and Cl ions. For the alkali-halogenids, such as NaCl or KCl, B&L found that $n \approx 9$. If $n = 9$ is used in Eq.(39) to determine theoretically the compression modulus κ, then the theoretical values of κ agree, in a first approximation, with the experimental values of κ, thus confirming the validity of the ansatz for the potential in Eq.(36).

We now apply Eq.(41) to the neutrino lattice of the elementary particles in order to determine the potential in the interior of the particles. We must use for r_0 the distance between two neighboring neutrinos in the lattice, which is equal to the range of the weak nuclear force. The range of the weak force is $1{\cdot}10^{-16}$ cm, as in Eq.(8), and so we have

$$r_0 = 1 \cdot 10^{-16}\,\mathrm{cm}\,. \tag{42}$$

We have used this value of r_0 throughout our explanation of the masses of the mesons and baryons, though r_0 was previously designated by the symbol a. We must, furthermore, replace e^2 by the interaction constant g_w^2 of the weak force which holds the nuclear lattice together. According to Perkins [22]

$$g_w^2 = 4\pi\hbar\,c \cdot 1.05 \cdot 10^{-2}\,(M_W/M_p)^2\,, \tag{43}$$

where M_W is the mass of the W boson, $M_W = 80.403\,\mathrm{GeV/c^2}$, and M_p is the mass of the proton, $M_p = 0.938\,272\,\mathrm{GeV/c^2}$. That means that

$$g_w^2 = 2.9976 \cdot 10^{-17}\,\mathrm{erg \cdot cm}. \tag{44}$$

We must also use the compression modulus κ of the nucleon. The value of the compression modulus of the nucleon has been determined theoretically by Bhaduri et al. [43], following earlier theoretical and experimental investigations of the compression moduli of nuclei. Bhaduri et al. found that the compression modulus $K_A(1)$ of the nucleon ranges from 900 to 1200 MeV, or is 940 MeV or 900 MeV for particular sets of parameters. We determine κ of the nucleon with

$$\kappa = 9/\rho_{\#}K_{\mathrm{nm}}\,, \tag{45}$$

from Eq.(1) in [43]. Bhaduri et al. write that "the compression modulus K_{nm} of nuclear matter is calculated by considering the nucleons as point particles", which they are not. Other assumptions are also sometimes made such as infinite nuclear matter, periodic boundary conditions, etc. Recent theoretical studies of the compressibility of "nuclear matter" [44,45,46] place

the compressibility K_{nm} at values from between 250 to 270 MeV, not much different from what is was twenty years earlier in [43]. Considering the uncertainty of K_{nm} it seems to be justified to set, in the case of the nucleon, $K_{nm} = K_A$, where K_A is defined as the compression modulus for a finite system with A nucleons. It then follows from Eq.(45) with the number density $\rho_{\#}$ being the number density per fm^3, and with the radius of the nucleon $R_0 = 0.88 \cdot 10^{-13}$ cm, and with $1\,\mathrm{MeV} = 1.6022 \cdot 10^{-6}$ erg that the compression modulus of the nucleon is

$$\kappa_n = 1.603 \cdot 10^{-35}\ \mathrm{cm}^2/\mathrm{dyn}\,, \tag{46}$$

if we use for $K_A(1)$ the value 1000 MeV. We will keep in mind that κ_n is not very accurate because $K_A(1)$ is not very accurate.

If we insert (42), (44), and (46) into $\mathrm{n} - 1 = 10.33\,\mathrm{r}_0^4/\mathrm{e}^2\kappa$ (Eq.41) we find an equation for the exponent n of the term $\mathrm{b}/\delta^{\mathrm{n}}$ in the repulsive part of the potential in a nuclear lattice,

$$\mathrm{n} = 1 + 2.164 \cdot 10^{-12} = 1 + \epsilon\,, \tag{47}$$

with $\mathrm{r}_0^4 = \mathrm{O}(10^{-64})$, $\mathrm{g}_w^2 = \mathrm{O}(10^{-17})$ and $\kappa = \mathrm{O}(10^{-35})$.

That means that

the potential ϕ in the interior of an elementary particle is given by

$$\phi = -\frac{\mathrm{a}}{\delta} + \frac{\mathrm{b}}{\delta^{1+\epsilon}} = \frac{\mathrm{a}}{\delta}\,[\frac{(\delta_0/\delta)^{\,\epsilon}}{\mathrm{n}} - 1\,]\,, \tag{48}$$

which we can reduce with $\mathrm{n} - 1 = \epsilon$, neglecting a term multiplied by $\epsilon^{\,2} = \mathrm{O}(10^{-24})$, using also $\mathrm{a} = 13.94\,\mathrm{e}^2$ (Eq.40) and $1/\mathrm{n} \cong (1 - \epsilon)$, to

$$\phi \cong -\frac{\mathrm{a}\,\epsilon}{\delta}\,[1 - \ln(\delta_0/\delta)] \cong -\frac{13.94\,\mathrm{g}_w^2\,\epsilon}{\delta}\,[1 - \ln(\delta_0/\delta)\,]\,. \tag{49}$$

In equilibrium the value of ϕ in the nuclear lattice is about $\mathrm{g}_w^2 \cdot \epsilon/\mathrm{e}^2 \approx 2.7 \cdot 10^{-10}$ times smaller than the corresponding electrostatic potential in an ionic lattice. A graph of the potential in Eq.(49) versus δ is shown in Fig. 6..

The minimum of the curve marks the equilibrium. From Eq.(49) follows with $\mathrm{F_w} = \mathrm{d}\phi/\mathrm{d}\delta$ and $\delta = 2\mathrm{r}$ that

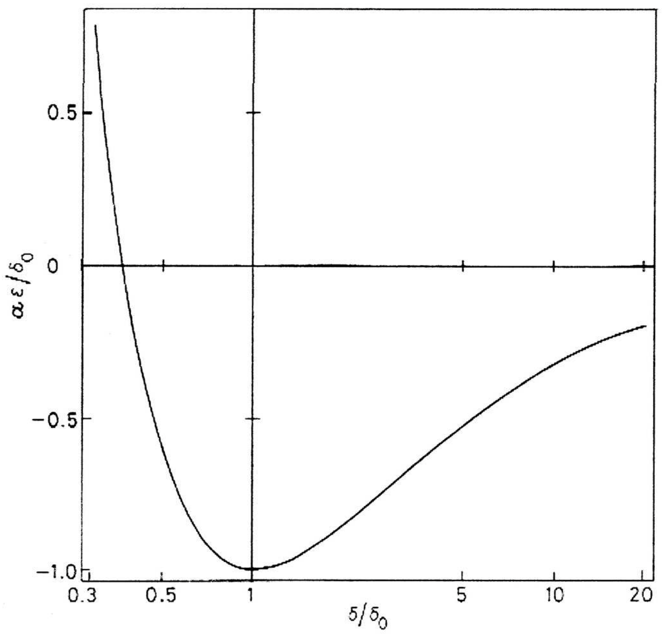

Fig.6: The potential ϕ of the weak force as a function of δ. After [23].

the weak force in the interior of the nuclear lattice is

$$\mathrm{F_w} \cong \frac{3.485 \cdot \mathrm{g}_w^2\,\epsilon}{\mathrm{r}^2} \cdot \ln(\frac{\delta}{\delta_0})\,. \qquad \text{(dyn)} \tag{50}$$

For all distances $\delta > \delta_0$ the force $\mathrm{F_w}$ is attractive, for all distances $\delta < \delta_0$ the force is repulsive. The small value of $\epsilon \approx 10^{-12}$ means that small displacements $\delta/\delta_0 < 1$ of the neutrinos from their equilibrium position, which carry the neutrinos into the domain of their neighboring neutrino, cause a very strong repulsive force between both neutrinos.

We have thus determined the potential of the weak force in the interior of the lattice of the elementary particles with lattice theory. Let us consider how this was done.

Following exactly the procedure used by B&L to determine the potential in the interior of an ionic crystal,

we have determined the potential in the interior of the lattice of an elementary particle

by using the parameters of the nuclear lattice. As in an ionic lattice the potential in a nuclear lattice has an attractive and a repulsive part, as is necessary for the stability of the lattice. We have found that in equilibrium in a nuclear lattice the absolute values of the attractive and repulsive terms of the potential are very nearly the same, because ϵ is on the order of 10^{-12}. One wonders, in view of the extraordinarily small value of ϵ, whether the potential in Eq.(48) is not an academic result. Therefore it has to be shown that Eq.(48) is indeed relevant for elementary particles. We can show that the mass of the muon neutrino $m(\nu_\mu)$ depends on $n - 1 = \epsilon = 2.164 \cdot 10^{-12}$ and yields a correct mass $m(\nu_\mu)$ only if $n - 1$ is on the order of 10^{-12}, for details see [9], Eq.(33) therein.

We have thus confirmed the validity of the potential $\phi = -\, a/\delta + b/\delta^n = -\, a/\delta + b/\delta^{1+\epsilon}$. That means *we have found the potential of the weak force which holds the nuclear lattice together.*

8 The strong force between two elementary particles

Crucial for the understanding of the existence of a strong force between the sides of two cubic lattices is the observation that

- *the sides of two halves of a cubic lattice cleaved in vacuum exert a strong, attractive force on each other.*

It is an automatic consequence of lattice theory that

- *the weak force, which holds the lattice together, is accompanied by a strong force which emanates from the sides of the lattice.*

This seems to contradict the simple observation that two salt crystals stacked upon each other can be separated without any effort. This is only so because the surface of a cubic crystal cleaved in air oxidizes immediately, and then the sides do not attract each other any longer. The origin of the force emanating from the sides of a cubic ionic lattice is the Coulomb force between ions of opposite polarity, i.e. the force which holds the lattice together. The

attractive force emanating from the side of a crystal cleaved in vacuum has a macroscopic value. The existence of this force has tangible consequences in space technology. The force between the sides of two cubic lattices was first studied by Born and Stern [47] (B&S).

If U_{12} is the potential between two halves of a crystal with the equal surfaces A, or the work that is necessary to separate the two halves of a cleaved crystal, then the capillary constant σ is given by Eq.(2) of B&S

$$\sigma = -\,\mathrm{U}_{12}/\mathrm{A}\,. \tag{51}$$

The capillary constant is, in the following, taken at zero degree absolute and against vacuum for the square outside area A of a cubic crystal. σ has been explained by B&S, but their formula cannot be used here because they use the value n = 9 of the alkali-halegonids. We will instead use Eq.(463) from Born [48] for the capillary constant σ_{100} of the (100) front surface of an ionic cubic crystal

$$\sigma_{100} = -\,\frac{\mathrm{e}^2}{\mathrm{r}_0^3}\,\frac{s_0(1)}{2}\cdot[\,1 - \frac{1}{\mathrm{n}}\,\frac{s_0(\mathrm{n})}{s_0(1)}\cdot\frac{S_0(1)}{S_0(\mathrm{n})}\,]\,. \tag{52}$$

The sums $\mathrm{s}_0(\mathrm{n})$ and $\mathrm{S}_0(\mathrm{n})$ originate from the contributions of the different lattice points to the repulsive part of the potential. The sign of the second term on the right hand side in Eq.(52) comes from the repulsive part of the potential in Eq.(36). For the capillary constant in a nuclear lattice we set $\mathrm{e}^2 = \mathrm{g}_w^2$, $\mathrm{n} = 1 + \epsilon$, (Eq.47), and $\mathrm{s}_0(1) = -\,0.0650$ from [48] p.743. We find that $\mathrm{s}_0(\mathrm{n}) \cong \mathrm{s}_0(1)$ since $\mathrm{n} = 1 + \epsilon$ and $\epsilon \cong 10^{-12}$. Similarly we have $\mathrm{S}_0(\mathrm{n}) \cong \mathrm{S}_0(1)$. Then we have

$$\sigma_{100} \cong \frac{0.065}{2}\,\frac{\mathrm{g}_w^2}{\mathrm{r}_0^3}\,\epsilon\,. \qquad (\mathrm{dyn/cm}) \tag{53}$$

The work required to separate one half of a nuclear lattice from the other half is according to Eq.(51) given by

$$\mathrm{U}_{12} \cong -\,\frac{0.065}{2}\,\frac{\mathrm{g}_w^2\,\epsilon}{\mathrm{r}_0^3}\cdot\mathrm{A}\,. \tag{54}$$

We determine the area A with the number of the lattice points in the cubic nuclear lattice

$$\mathrm{N} = 2.854\cdot 10^9\,, \tag{55}$$

from Eq.(17). It follows that $A = (\sqrt[3]{N} \cdot r_0)^2$. And it follows that the strong attractive force between the sides of two nuclear lattices is

$$F_s = \frac{dU_{12}}{dr} = -\frac{d\sigma}{dr} \cdot A = \frac{3 \cdot 0.065\, g_w^2\, \epsilon}{2\, r^4} \cdot A\,. \tag{56}$$

The force emanating from the front surface of a cubic nuclear lattice, the strong force, is

$$F_s = \frac{0.0975\ g_w^2\ \epsilon}{r^4} \cdot (\sqrt[3]{N} \cdot r_0)^2\,. \qquad \text{(dyn)} \tag{57}$$

The strong force depends first of all on the *weak interaction constant* g_w^2, and secondly on the number of lattice points on the side of the lattice, $(\sqrt[3]{N})^2 = 2.012 \cdot 10^6$. The strong force is also inversely proportional to the *fourth* power of the distance between the particles. In our model the strong force depends, other than on constants and r^{-4}, on the number $N^{2/3}$ of the lattice points at the side of the lattice. That means that the force which emanates from the sides of the π mesons is the same as the force which emanantes from the sides of the proton, because both have, in this model, the same number of lattice points. We also note that the strong force, (Eq.57), is independent of a charge. The entire force which goes out from the surface of the lattice is six times as much as given by Eq.(57).

The ratio of the strong force $6\,F_s$ emanating from the entire surface of a cubic nuclear lattice to the weak force F_w, Eq.(50), between the neutrinos in the same lattice is

$$6\,F_s = \frac{0.338 \cdot 10^6}{(r/r_0)^2 \ln(r/r_0)} \cdot F_w\,. \tag{58}$$

The prime factor in the ratio of the strong and weak forces is the number of lattice points at a side of the lattice $(\sqrt[3]{N})^2 = 2.012{\cdot}10^6$. The ratio F_s/F_w is very sensitive to the ratio r/r_0. For $r > r_0$ the strong force decreases with increasing r, for $r \to r_0$ we have $F_s \to \infty$, and for $r < r_0$ the strong force is repulsive, as it must be when one lattice enters another lattice.

To summarize: According to lattice theory

- *the existence of the strong nuclear force between two elementary particles is an automatic consequence of our explanation of the masses of the elementary particles with cubic nuclear lattices,*

held together by the weak nuclear force. The lattices we have used for the explanation of the masses of the particles consist of photons or neutrinos. That means:

We do not use hypothetical particles.

We have found long sought after answers to the questions what is the weak nuclear force and the strong nuclear force, and why is the strong force so much stronger than the weak force? The strong force is nothing but the sum of the large number of unsaturated weak forces at the surface of the nuclear lattice.

In order to understand the origin of the strong nuclear force one has to understand the structure of the elementary particles, which we have explained with nuclear lattices. We have now also understood the strength of the weak force which holds the nuclear lattice together and the cause of the strong force between two nuclear lattices.

9 The rest mass of the muon

Surprisingly one can also explain the mass of the $\mu^{\pm}$ mesons with our explanation of the $\pi^{\pm}$ mesons. The existence of the muons has been a puzzle since their discovery 70 years ago. The muons belong to the lepton family which is distinguished from the mesons and baryons by the absence of strong interaction with the mesons and baryons. The charged leptons make up 1/2 of the number of stable elementary particles. The standard model of the particles does not deal with the lepton masses. Barut [49] has given a simple and quite accurate empirical formula relating the masses of the electron, muon and τ lepton, which formula has been extended by Gsponer and Hurni [50] to the quark masses.

The origin of most of the $\mu^{\pm}$ mesons is the decay of the $\pi^{\pm}$ mesons, $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$, or the decay of the $\mathrm{K}^{\pm}$ mesons, $\mathrm{K}^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$. The mass of the muons is $\mathrm{m}(\mu^{\pm}) = 105.658\,369 \pm 9{\cdot}10^{-6}\,\mathrm{MeV/c^2}$, according to the Review of Particle Physics [2]. The mass of the muons is usually compared to the mass of the electron and is very often said to be

$$\mathrm{m}(\mu^{\pm}) = \mathrm{m}(\mathrm{e}^{\pm}) \cdot (1 + 3/2\alpha_f) = 206.554\,\mathrm{m}(\mathrm{e}^{\pm}) = 0.99896\,\mathrm{m}(\mu^{\pm})(exp)\,, \quad (59)$$

(with the fine structure constant $\alpha_f = 1/137.036$). The experimental value of $\mathrm{m}(\mu^{\pm})$ is $206.768\,\mathrm{m}(\mathrm{e}^{\pm})$. This formula for $\mathrm{m}(\mu^{\pm})$ was given by Barut [51]

following an earlier suggestion by Nambu [10] that the mass of the π meson is $\approx 2/\alpha_f \cdot \mathrm{m(e^{\pm})}$ and that $\mathrm{m}(\mu^{\pm}) \approx 3/2\alpha_f \cdot \mathrm{m(e^{\pm})}$. The muons are "stable", their lifetime $\tau(\mu^{\pm}) = 2.19703 \cdot 10^{-6} \pm 4 \cdot 10^{-11}$ sec is about a hundred times longer than the lifetime of the $\pi^{\pm}$ mesons, that means longer than the lifetime of any other elementary particle, but for the electrons, protons and neutrons.

Comparing the mass of the $\mu^{\pm}$ mesons to the mass of the $\pi^{\pm}$ mesons from which the $\mu^{\pm}$ mesons emerge we find, with $\mathrm{m}(\pi^{\pm}) = 139.570\,18\,\mathrm{MeV/c^2}$, that

$$\begin{aligned} \mathrm{m}(\mu^{\pm})/\mathrm{m}(\pi^{\pm}) &= 0.757027 = 1.00937 \cdot 3/4 \\ &\cong 3/4 + \alpha_f = 0.757297 \\ &= 1.00036 \cdot \mathrm{m}(\mu^{\pm})/\mathrm{m}(\pi^{\pm})(\mathit{exp})\,. \end{aligned} \tag{60}$$

The term $+\alpha_f$ in the preceding equation will be explained later, Eq.(100). The mass of the $\mu^{\pm}$ mesons is in a good approximation 3/4 of the mass of the $\pi^{\pm}$ mesons. We have also $\mathrm{m}(\pi^{\pm}) - \mathrm{m}(\mu^{\pm}) = 33.9118\,\mathrm{MeV/c^2} = 0.24297\mathrm{m}(\pi^{\pm})$ or approximately $1/4 \cdot \mathrm{m}(\pi^{\pm})$. The mass of the electron is approximately 1/207 of the mass of the muon, the contribution of $\mathrm{m(e^{\pm})}$ to $\mathrm{m}(\mu^{\pm})$ will therefore be neglected in the following. We assume, as we have done before and as appears to be natural, that the particles, including the muons, *consist of the particles into which they decay.* The $\mu^{\pm}$ mesons decay via $\mu^{\pm} \rightarrow \mathrm{e^{\pm}} + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e)$ ($\approx$ 100%). The muons are apparently composed of an elementary electric charge $\mathrm{e^{\pm}}$ and some of the neutrinos, antineutrinos and their oscillations which are, according to Section 6, present in the cubic neutrino lattice of the $\pi^{\pm}$ mesons from which the $\mu^{\pm}$ mesons come. The $\mu^{\pm}$ mesons with a mass $\mathrm{m}(\mu^{\pm}) \cong 3/4 \cdot \mathrm{m}(\pi^{\pm})$ seem to be related to the $\pi^{\pm}$ mesons rather than to the electron with which the $\mu^{\pm}$ mesons have been compared traditionally, although $\mathrm{m}(\mu^{\pm})$ is separated from $\mathrm{m(e^{\pm})}$ by a factor $\cong 207$.

From Eq.(33) followed that the rest mass of a muon neutrino should be about 50 milli-eV/$\mathrm{c^2}$. Provided that the mass of an electron neutrino $\mathrm{m}(\nu_e)$ is small as compared to $\mathrm{m}(\nu_\mu)$, as will be shown by Eq.(72), we find, with N $= 2.854 \cdot 10^9$, that:

(a) The difference of the rest masses of the $\mu^{\pm}$ and $\pi^{\pm}$ mesons is nearly equal to the sum of the rest masses of all muon neutrinos, respectively anti-muon neutrinos, in the $\pi^{\pm}$ mesons.

$$\mathrm{m}(\pi^{\pm}) - \mathrm{m}(\mu^{\pm}) = 33.912\,\mathrm{MeV/c^2} \quad \text{versus} \quad \mathrm{N'}/4 \cdot \mathrm{m}(\nu_\mu) \approx 35.68\,\mathrm{MeV/c^2}\,.$$

(b) The energy in the oscillations of all $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos in the $\pi^\pm$ mesons is nearly the same as the energy in the oscillations of all $\bar{\nu}_\mu, \nu_e, \bar{\nu}_e$, respectively $\nu_\mu, \bar{\nu}_e, \nu_e$, neutrinos in the $\mu^\pm$ mesons. The oscillation energy is the rest mass of a particle minus the sum of the rest masses of all neutrinos in the particle as in Eq.(33). With $\mathrm{m}(\nu_\mu) = \mathrm{m}(\bar{\nu}_\mu)$ and $\mathrm{m}(\nu_e) = \mathrm{m}(\bar{\nu}_e)$ from Eqs.(68,71) we have

$$\mathrm{E}_\nu(\pi^\pm) = \mathrm{m}(\pi^\pm)\mathrm{c}^2 - \mathrm{N}'/2 \cdot [\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_e)]\mathrm{c}^2 = 68.22\,\mathrm{MeV} \qquad (61)$$

versus

$$\mathrm{E}_\nu(\mu^\pm) = \mathrm{m}(\mu^\pm)\mathrm{c}^2 - \mathrm{N}'/4 \cdot \mathrm{m}(\nu_\mu)\mathrm{c}^2 - \mathrm{N}'/2 \cdot \mathrm{m}(\nu_e)\mathrm{c}^2 = 69.98\,\mathrm{MeV}\,. \qquad (62)$$

Equation (62) means that either all $\mathrm{N}'/4$ muon neutrinos or all $\mathrm{N}'/4$ anti-muon neutrinos have been removed from the $\pi^\pm$ lattice in its decay. If, e.g., any ν_μ neutrinos were to remain in the μ^+ meson after the decay of the π^+ meson they ought to appear in the decay of μ^+, but they do not.

We attribute the 1.768 MeV difference between the left and right side of (a) to the second order effects which cause the deviations of the masses of the particles from the integer multiple rule. There is also the difference that the left side of (a) deals with two charged particles, whereas the right side deals with neutral particles. (b) seems to say that the oscillation energy of all neutrinos in the $\pi^\pm$ lattice is conserved in the $\pi^\pm$ decay, which seems to be necessary because the oscillation frequencies in $\pi^\pm$ and $\mu^\pm$ must follow Eq.(14) as dictated by the group velocity limitation. If indeed

$$\mathrm{E}_\nu(\pi^\pm) = \mathrm{E}_\nu(\mu^\pm) \qquad (63)$$

then it follows from the difference of Eqs.(61) and (62) that

$$\mathrm{m}(\pi^\pm) \,-\, \mathrm{m}(\mu^\pm) = \mathrm{N}'/4 \cdot \mathrm{m}(\nu_\mu) = \mathrm{N}'/4 \cdot \mathrm{m}(\bar{\nu}_\mu)\,. \qquad (64)$$

The $\mathrm{N}'/4$ electron neutrinos, respectively anti-electron neutrinos, which come, as we will see in Section 11, into the $\mu^\pm$ mesons with the elementary electric charge $\mathrm{e}^\pm$, are the recipients of 1/4 of the oscillation energy $\mathrm{E}_\nu(\pi^\pm)$ of the $\pi^\pm$ mesons which becomes available when the muon neutrinos or anti-muon neutrinos leave the $\pi^\pm$ lattice in the $\pi^\pm$ decay. The neutrinos coming with $\mathrm{e}^\pm$ into $\mu^\pm$ make it possible that $\mathrm{E}_\nu(\pi^\pm) = \mathrm{E}_\nu(\mu^\pm)$. After the $\pi^\pm$ decay the muon neutrinos in $\mu^\pm$ retain their original oscillation energy $\mathrm{E}_\nu(\pi^\pm)/4$,

the electron neutrinos and anti-electron neutrinos in $\mu^\pm$ retain their original oscillation energies $\mathrm{E}_\nu(\pi^\pm)/4$ as well. The remaining oscillation energy $\mathrm{E}_\nu(\pi^\pm)/4$ of the $\pi^\pm$ lattice so far not accounted for is picked up by the electron neutrinos, respectively anti-electron neutrinos, brought into $\mu^\pm$ by the elementary electric charge. Without a recipient for this oscillation energy $\mathrm{E}_\nu(\pi^\pm)/4$ a stable new particle can apparently not be formed in the $\pi^\pm$ decay, that means there is no μ^0 meson.

We should note that in the $\pi^\pm$ decays only *one single* muon neutrino or single anti-muon neutrino is emitted, not $\mathrm{N}'/4$ of them, but that in the $\pi^\pm$ decay 33.912 MeV are released. Since according to (b) the oscillation energy of the neutrinos in the $\pi^\pm$ mesons is conserved in their decay the 33.912 MeV released in the $\pi^\pm$ decay can come from *no other source* then from the rest masses of the muon neutrinos or the anti-muon neutrinos in the $\pi^\pm$ mesons. The average kinetic energy of the neutrinos in the $\pi^\pm$ lattice is about 50 milli-eV, so it is not possible that a single neutrino in $\pi^\pm$ possesses an energy of 33.9 MeV. The 33.9 MeV can come only from the sum of the muon neutrino or anti-muon neutrino rest masses in the $\pi^\pm$ mesons. However, what happens then to the neutrino numbers?

Either conservation of neutrino numbers is violated or the decay energy comes from equal numbers of muon neutrinos and anti-muon neutrinos. Equal numbers $\mathrm{N}'/8$ muon neutrinos and $\mathrm{N}'/8$ anti-muon neutrinos would then be in the $\mu^\pm$ mesons. This would not make a difference in either the oscillation energy or in the sum of the rest masses of the neutrinos or in the spin of the $\mu^\pm$ mesons. A situation similar to the $\pi^\pm$ decay occurs in the $\mu^\pm$ decay. The 105.147 MeV released in the $\mu^\pm$ decay comes, in our model, mainly from the energy in the rest masses of either $\mathrm{N}'/4{\cdot}\nu_\mu$ or $\mathrm{N}'/4{\cdot}\bar{\nu}_\mu$ neutrinos and their oscillations, because the rest masses of the $\nu_e, \bar{\nu}_e$ neutrinos, which are also in $\mu^\pm$, are so small. Conservation of neutrino numbers in the $\mu^\pm$ decay requires that $\mathrm{N}'/8$ muon neutrinos and $\mathrm{N}'/8$ anti-muon neutrinos are in the $\mu^\pm$ lattice. Since $\mathrm{m}(\nu_\mu) = \mathrm{m}(\bar{\nu}_\mu)$ we will, however, for the sake of simplicity, use the term $\mathrm{N}'/4 \cdot \mathrm{m}(\nu_\mu)$ or $\mathrm{N}'/4 \cdot \mathrm{m}(\bar{\nu}_\mu)$ for $\mathrm{N}'/8 \cdot [\mathrm{m}(\nu_\mu) + \mathrm{m}(\bar{\nu}_\mu)]$.

Inserting $\mathrm{m}(\pi^\pm) - \mathrm{m}(\mu^\pm) = \mathrm{N}'/4 \cdot \mathrm{m}(\nu_\mu)$ from Eq.(64) into Eq.(62) we arrive at an equation for *the theoretical value of the mass of the $\mu^\pm$ mesons.* It is

$$\mathrm{m}(\mu^\pm)\mathrm{c}^2 = 1/2 \cdot [\, \mathrm{E}_\nu(\pi^\pm) + \mathrm{m}(\pi^\pm)\mathrm{c}^2 + \mathrm{N}'\,\mathrm{m}(\nu_e)\mathrm{c}^2/2\,] = 103.95\,\mathrm{MeV}\ , \quad (65)$$

which is $0.9838\,\mathrm{m}(\mu^\pm)\mathrm{c}^2(exp)$ and expresses $\mathrm{m}(\mu^\pm)$ through the well-known

mass of $\pi^\pm$, the calculated oscillation energy of $\pi^\pm$, and a small contribution (0.4%) of the electron neutrino and anti-electron neutrino masses. Eq.(65) shows that our explanation of the mass of the $\mu^\pm$ mesons comes close to the experimental value $\mathrm{m}(\mu^\pm) = 105.658\,\mathrm{MeV/c^2}$. With $\mathrm{E}_\nu(\pi^\pm) = \mathrm{E}_\nu(\mu^\pm)$ and with $\mathrm{m}(\pi^\pm)$ from Eq.(34) we find a different form of Eq.(65) which is, with $\mathrm{m}(\nu_e) = \mathrm{m}(\bar{\nu}_e)$, in the case of the μ^+ meson,

$$\mathrm{m}(\mu^+) = \mathrm{E}_\nu(\mu^\pm)/\mathrm{c}^2 + \mathrm{N}'\mathrm{m}(\bar{\nu}_\mu)/4 + \mathrm{N}'\mathrm{m}(\nu_e)/2\ . \tag{66}$$

As Eq.(66) tells, the rest mass of the muons is the sum of the rest masses of the muon neutrinos, respectively anti-muon neutrinos, and of the masses of the electron neutrinos and anti-electron neutrinos which are in the muon lattice, plus the oscillation energy of these neutrinos, neglecting the mass of $\mathrm{e}^\pm$. The ratio $\mathrm{m}(\mu^\pm)/\mathrm{m}(\pi^\pm)$ is 3/4, as it must be, if we divide Eq.(66) by $\mathrm{m}(\pi^\pm)$ which follows from Eq.(34) and if we neglect the small masses of the electron neutrinos and anti-electron neutrinos.

The $\mu^\pm$ mesons cannot be point particles because they have a neutrino lattice. The commonly held belief that the $\mu^\pm$ mesons are point particles is based on the results of scattering experiments. But at a true point the density of a "point particle" would be infinite, which poses a problem. It is odd that the $\mu^\pm$ mesons, which emerge from the $\pi^\pm$ mesons and have a mass which is $3/4 \cdot \mathrm{m}(\pi^\pm)$, should have a mass which is concentrated in a point, whereas it is accepted and measured that the $\pi^\pm$ mesons have a body of finite size. Since, on the other hand, neutrinos do not interact, in a very good approximation, with charge or mass it will not be possible to determine the size of the $\mu^\pm$ meson lattice through conventional scattering experiments. The $\mu^\pm$ mesons *appear* to be point particles because only the electric charge of the muons participates in the scattering process and the elementary electric charge scatters like a point particle. *99.5% of the muon mass consists of non-interacting neutrinos.* There is therefore currently no measurable difference in e^--p and μ^--p scattering.

Finally we must address the question for what reason do the muons or leptons not interact strongly with the mesons and baryons? We have shown in Section 8 that a strong force emanates from the sides of a cubic lattice caused by the unsaturated weak forces of about 10^6 lattice points at the surface of the lattice of the mesons and baryons. This follows from the study of Born and Stern [47] which dealt with the forces between two parts of a cubic lattice cleaved in vacuum. The strong force between two particles is an

automatic consequence of the weak internal force which holds the particles together. If the muons have a lattice consisting of N/8 muon neutrinos and N/8 anti-muon neutrinos and of N/4 electron neutrinos and N/4 anti-electron neutrinos their lattice surface is not the same as the surface of the cubic ν_μ, $\bar{\nu}_\mu$, ν_e, $\bar{\nu}_e$ lattice of the mesons and baryons. Therefore the muon lattice does not bind with the cubic lattice of the mesons and baryons.

To summarize what we have learned about the $\mu^\pm$ mesons. Eq.(66) says that the energy in $\mathrm{m}(\mu^\pm)\mathrm{c}^2$ is the sum of the oscillation energies plus the sum of the energy of the rest masses of the neutrinos and antineutrinos in $\mathrm{m}(\mu^\pm)$, neglecting the energy in $\mathrm{e}^\pm$. The three neutrino types in the lattice of the $\mu^\pm$ mesons shown in Fig. 7 are the remains of the cubic neutrino lattice in the $\pi^\pm$ mesons. Since $\mathrm{N}/8 \cdot \nu_\mu$ and $\mathrm{N}/8 \cdot \bar{\nu}_\mu$ neutrinos have been removed from the $\pi^\pm$ lattice in the $\pi^\pm$ decay the rest mass of the $\mu^\pm$ mesons must be $\cong 3/4 \cdot \mathrm{m}(\pi^\pm)$, as the experiments find. The $\mu^\pm$ mesons are not point particles.

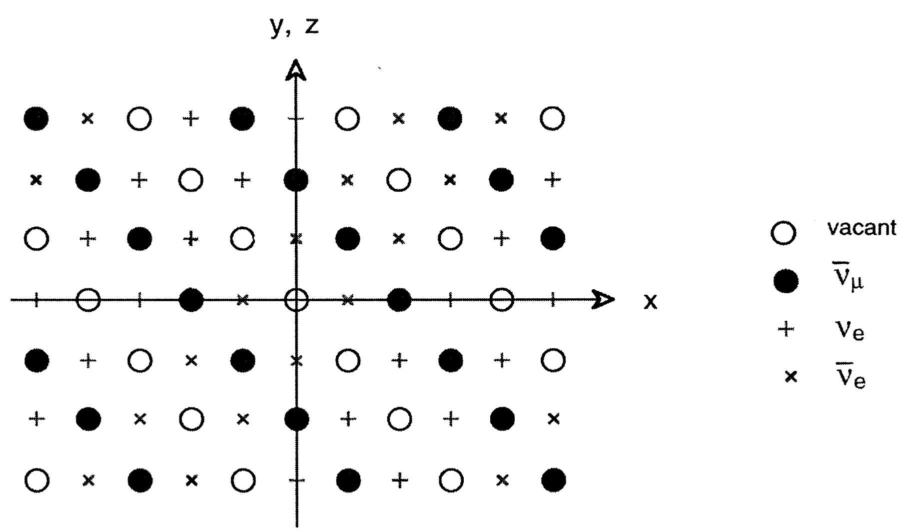

Fig. 7: A section through the central part of the neutrino lattice of the μ^+ meson without its charge.

The mass of the $\tau^\pm$ leptons, whose mass is $\mathrm{m}(\tau^\pm) = 1776.99\,\mathrm{MeV/c^2} = 16.8182\,\mathrm{m}(\mu^\pm)$ or $1.8939\,\mathrm{m(p)}$, follows from the most frequent leptonic decay of the $\mathrm{D}_s^\pm$ mesons, $\mathrm{D}_s^\pm \rightarrow \tau^\pm + \nu_\tau(\bar{\nu}_\tau)$ (6.4%). It can be shown readily that the oscillation energies of the lattices in $\mathrm{D}_s^\pm$ and in $\tau^\pm$ are the same. From

that follows that the energy in the rest mass of the $\tau^{\pm}$ leptons is the sum of the oscillation energy in the τ lattice plus the sum of the energy of the rest masses of all neutrinos and antineutrinos in the τ lattice, just as with the $\mu^{\pm}$ mesons. We will skip the details. The tau leptons are not point particles either.

10 The neutrino masses

Now we come to the neutrino masses. As mentioned before, the experiments show that the electron neutrinos, muon neutrinos and tau neutrinos are different. It is undisputed that all of them have no charge and the same spin. However, if the three neutrino types have different rest masses then the electron neutrinos, muon neutrinos and tau neutrinos are different. There is no certain knowledge what the neutrino masses are. Numerous values for $m(\nu_e)$ and $m(\nu_\mu)$ have been proposed and upper limits for them have been established experimentally which have, with time, decreased steadily. The Review of Particle Physics [2] gives for the mass of the electron neutrino the value $< 2\,\mathrm{eV/c^2}$. Neither the Superkamiokande [37] nor the Sudbury [38] experiments determine a neutrino mass, however, both experiments make it very likely that the neutrinos have rest masses. We will now determine the neutrino masses from the composition of the $\pi^{\pm}$ mesons and from the β-decay of the neutron.

If the same principle that applies to the decay of the $\pi^{\pm}$ mesons, namely that in the decay the oscillation energy of the decaying particle is conserved and that an entire neutrino type supplies the energy released in the decay, as expressed by Eqs.(61,62), also applies to the decay of the neutron $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$, then the mass of the anti-electron neutrino can be determined from the known difference $\Delta = \mathrm{m(n)} - \mathrm{m(p)} = 1.293\,332\,\mathrm{MeV/c^2} \cong 5/2 \cdot \mathrm{m(e)/c^2}$ [2]. Nearly one half of Δ comes from the energy lost by the emission of the electron, whose mass is $0.510\,9989\,\mathrm{MeV/c^2}$. N anti-electron neutrinos are in the neutrino quadrupoles in the neutron, one-fourth of them is carried away by the emitted electron. We have seen in the paragraph above Fig. 4 that the decay sequence of the $\pi^{\pm}$ mesons requires that the electron carries with it $\mathrm{N}'/4$ electron neutrinos, if the $\pi^{\pm}$ mesons consist of a lattice with a center neutrino or antineutrino and equal numbers of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ neutrinos as required by conservation of neutrino numbers during the creation of $\pi^{\pm}$. The composition of the electron will be discussed in detail in Section 11.

The electron can carry $N'/4$ anti-electron neutrinos as well as $N'/4$ electron neutrinos, see Appendix C. Since, as we will see shortly, $m(\nu_e) = m(\bar{\nu}_e)$ this does not make a difference energetically but is relevant for the orientation of the spin vector of the emitted electron. After the neutron has lost $N'/4$ anti-electron neutrinos to the electron emitted in the β-decay the other $3/4 \cdot N'$ anti-electron neutrinos in the neutron provide the energy $(\Delta - m(e^-))c^2 = 0.782\,333\,\mathrm{MeV} \cong 3/2 \cdot m(e)c^2$ released in the decay of the neutron. From this follows, after division by $3/4 \cdot N'$, that the rest mass of the anti-electron neutrino is

$$m(\bar{\nu}_e) = 0.365\,\text{milli-eV/c}^2\,. \tag{67}$$

Since theoretically the anti-neutron decays as $\bar{n} \rightarrow \bar{p} + e^+ + \nu_e$ it follows from the same considerations as with the decay of the neutron that

$$m(\nu_e) = m(\bar{\nu}_e)\,. \tag{68}$$

We note that

$$N'/4 \cdot m(\nu_e) = N'/4 \cdot m(\bar{\nu}_e) = 0.51\,m(e^\pm)\,. \tag{69}$$

This equation is, as we will see, fundamental for the explanation of the mass of the electron.

Inserting Eq.(67) into Eq.(33) for the sum of the masses of the neutrinos in $\pi^\pm$ we find that

$$m(\nu_\mu) = 49.91\,\text{milli-eV/c}^2\,. \tag{70}$$

Since the same considerations apply for either the π^+ or the π^- meson it follows that

$$m(\nu_\mu) = m(\bar{\nu}_\mu)\,. \tag{71}$$

Experimental values for the rest masses of the different neutrino types are not available. However, it appears that for the $\nu_\mu \leftrightarrow \nu_e$ oscillation the value for $\Delta m^2 = m_2^2 - m_1^2 = 3.2 \times 10^{-3}\,\mathrm{eV}^2$ given on p.1565 of [37] can be used to determine $m_2 = m(\nu_\mu)$ if $m_1 = m(\nu_e)$ is much smaller than m_2. We have then $m(\nu_\mu) \approx 56.56\,\text{milli-eV/c}^2$, which is compatible with the value of $m(\nu_\mu)$ given in Eq.(70).

From Eqs.(67,70) follows that

$$m(\nu_e) = 1/136.74 \cdot m(\nu_\mu) \cong \alpha_f m(\nu_\mu)\,. \tag{72}$$

$1/136.74$ is 1.0021 times the fine structure constant $\alpha_f = e^2/\hbar c = 1/137.036$. It does not seem likely that Eq.(72) is just a coincidence. The probability

for this being a coincidence is zero considering the infinite pool of numbers on which the ratio $\mathrm{m}(\nu_e)/\mathrm{m}(\nu_\mu)$ could settle.

The mass of the τ neutrino ν_τ can be determined from the decay $\mathrm{D}_s^\pm \rightarrow \tau^\pm + \nu_\tau\,(\bar{\nu}_\tau)$, and the subsequent decay $\tau^\pm \rightarrow \pi^\pm + \bar{\nu}_\tau(\nu_\tau)$, stated in [2]. The appearance of ν_τ in the decay of $\mathrm{D}_s^\pm$ and the presence of $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos in the $\pi^\pm$ decay product of the $\tau^\pm$ leptons means that $\nu_\tau, \bar{\nu}_\tau, \nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos must be in the $\mathrm{D}_s^\pm$ lattice. The additional ν_τ and $\bar{\nu}_\tau$ neutrinos can be accomodated in $\mathrm{D}_s^\pm$ by a body-centered cubic lattice, in which there is in the center of each cubic cell one particle different from the particles in the eight cell corners (Fig. 5). In a body-centered cubic lattice are $\mathrm{N}'/4$ cell centers, if N' is the number of lattice points without the cell centers. If the particles in the cell centers are tau neutrinos then $\mathrm{N}'/8$ ν_τ neutrinos and $\mathrm{N}'/8$ anti-tau neutrinos $\bar{\nu}_\tau$ must be present, because of conservation of neutrino numbers. From $\mathrm{m}(\mathrm{D}_s^\pm) = 1968.2\,\mathrm{MeV/c}^2$ and $\mathrm{m}(\tau^\pm) = 1777\,\mathrm{MeV/c}^2$ follows that

$$\mathrm{m}(\mathrm{D}_s^\pm) - \mathrm{m}(\tau^\pm) = 191.2\,\mathrm{MeV/c}^2 = \mathrm{N}'/8 \cdot \mathrm{m}(\nu_\tau)\,. \tag{73}$$

The rest mass of the τ neutrinos is therefore

$$\mathrm{m}(\nu_\tau) = \mathrm{m}(\bar{\nu}_\tau) = 0.536\,\mathrm{eV/c}^2\,. \tag{74}$$

From the neutrino masses given by Eq.(74) and Eq.(70) follows that

$$\mathrm{m}(\nu_\tau) = 10.76\,\mathrm{m}(\nu_\mu) = 1.048\,(\alpha_w/\alpha_f)\mathrm{m}(\nu_\mu)\,, \tag{75}$$

where α_w is the weak coupling constant $\alpha_w = g_w^2/4\pi\hbar\mathrm{c}$, (Eq.43), and α_f is the fine structure constant. We keep in mind that g_w^2 in α_w is not nearly as accurately known as e^2 in α_f. With Eq.(72) we find that

$$\mathrm{m}(\nu_\tau) = 1.048 \cdot (\alpha_w/\alpha_f^2) \cdot \mathrm{m}(\nu_e) = 1474\,\mathrm{m}(\nu_e)\,. \tag{76}$$

According to Eqs.(67,70,74) the sum of the masses of the electron neutrino, muon neutrino and tau neutrino is $0.586\,\mathrm{eV/c}^2$, primarily because of the mass of the tau neutrino. According to the Review of Particle Physics (2004, p.439) it follows from astrophysical data that the sum of the neutrino masses $\sum_i \mathrm{m}(\nu_i) \leq 0.7\,\mathrm{eV/c}^2$. We arrive at essentially the same result.

To summarize what we have learned about the masses of the leptons: We have found an explanation for the mass of the $\mu^\pm$ mesons and $\tau^\pm$ leptons. We have also determined the rest masses of the e, μ, τ neutrinos and antineutrinos. In other words, we have found the masses of all leptons, exempting the electron, which will be dealt with in the next Section.

11 The electron

The electron differs from the other particles we have considered in so far as it appears that its electric charge cannot be separated from its mass, whereas in the other charged particles the mass of the electric charge is, in a first approximation, unimportant for the mass of the particles. Even in the muons the mass of the electron contributes only five thousandth of the muon mass. On the other hand, the electron is fundamental for the stability of the charged particles whose lifetime is sometimes orders of magnitude larger than the lifetime of their neutral counterparts. For example the lifetime of the $\pi^{\pm}$ mesons is eight orders of magnitude larger than the lifetime of π^0, the lifetime of the proton is infinite, whereas the neutron decays in about 900 seconds and, as a startling example, the lifetime of $\Sigma^{\pm}$ is $O(10^{-10})$ seconds, whereas the lifetime of Σ^0 is $O(10^{-20})$ seconds. There is something particular to the interaction of the elementary electric charge with the particle masses.

After J.J. Thomson [52] discovered the small corpuscle which soon became known as the electron an enormous amount of theoretical work has been done to explain the existence of the electron. Some of the most distinguished physicists have participated in this effort. Lorentz [53], Poincaré [54], Ehrenfest [55], Einstein [56], Pauli [57], and others showed that it is fairly certain that the electron cannot be explained as a purely electromagnetic particle. In particular it was not clear how the elementary electric charge could be held together in its small volume because the internal parts of the charge repel each other. Poincaré [58] did not leave it at showing that such an electron could not be stable, but suggested a solution for the problem by introducing what has become known as the Poincaré stresses whose origin however remained unexplained. These studies were concerned with the static properties of the electron, its mass m(e) and its electric charge e; the positron, the spin and the neutrinos were not known at that time. In order to explain the electron with its existing mass and charge it appears to be necessary to add to Maxwell's equations a non-electromagnetic mass and a non-electromagnetic force which could hold the electric charge together. We shall see what this mass and force is.

The discovery of the spin of the electron by Uhlenbeck and Goudsmit [59] increased the difficulties of the problem in so far as it now had also to be explained how the angular momentum $\hbar/2$ and the magnetic moment μ_e come about. The spin of a point-like electron seemed to be explained by Dirac's [60] equation, however it turned out later [61] that Dirac type equations can

be constructed for any value of the spin. Afterwards Schrödinger [62] tried to explain the spin and the magnetic moment of the electron with the so-called Zitterbewegung. Dirac [63] suggested a model of an electron without spin, consisting of a charged, hollow sphere held together by surface tension. The first higher mode of oscillation appeared to be the muon, to quote “one can look upon the muon as an electron excited by radial oscillations”. Many other models of the electron were proposed. On p.74 of his book “The Enigmatic Electron” Mac Gregor [64] lists more than thirty such models. At the end none of these models has been successful because the problem developed a seemingly insurmountable difficulty when it was shown through electron scattering experiments that the charge radius of the electron must be smaller than 10^{-16} cm [65], in other words that the electron appears to be a point particle, at least by three orders of magnitude smaller than the classical electron radius $r_e = e^2/m_e c^2 = 2.8179 \cdot 10^{-13}$ cm. This, of course, makes it very difficult to explain how a particle can have a finite angular momentum when the radius goes to zero, and how an electric charge can be confined in an infinitesimally small volume. If the elementary electric charge would be in a volume with a radius of $O(10^{-16})$ cm the Coulomb self-energy would be orders of magnitude larger than the rest mass of the electron, which is not realistic. The choice is between a massless point charge and a finite size particle with a non-interacting mass to which an elementary electric charge is attached. It seems fair to say that at present, more than 100 years after the discovery of the electron, we do not have an accepted theoretical explanation of the electron.

We propose in the following, as in [66], that the non-electromagnetic mass which seems to be necessary in order to explain the electron consists of neutrinos. This is actually a necessary consequence of our model of the mass of the $\pi^\pm$ mesons and of the decay sequence of $\pi^\pm$. And we propose that the non-electromagnetic force required to hold the electric charge and the neutrinos in the electron together is the weak nuclear force which, as we have suggested, holds together the masses of the mesons and baryons and also the mass of the muons. Since the range of the weak nuclear force is on the order of 10^{-16} cm the neutrinos must be arranged in a lattice with the weak force extending from each lattice point only to the nearest neighbors. The $O(10^{-13})$ cm size of the neutrino lattice in the electron does not at all contradict the results of the scattering experiments, just as the explanation of the mass of the muons with our model does not contradict the apparent point particle characteristics of the muon, because neutrinos are in a very good

approximation non-interacting and therefore are not noticed in scattering experiments with electrons.

The rest mass of the electron is $\mathrm{m(e)} = 0.510\,998\,92 \pm 4{\cdot}10^{-8}\,\mathrm{MeV/c^2}$ and the electrostatic charge of the electron is $\mathrm{e} = 4.803\,2044 \cdot 10^{-10}\,\mathrm{esu}$, as stated in the Review of Particle Physics [2]. *The objective of a theory of the electron must, first of all, be the explanation of* $\mathrm{m(e^{\pm})}$ *and* $\mathrm{e^{\pm}}$, but also of $\mathrm{s(e^{\pm})}$ and of the magnetic moment μ_e. We will first explain the rest mass of the electron, making use of what we have learned about the explanation of the mass of the $\mu^{\pm}$ mesons in Section 9. The muons are leptons, just as the electrons, that means that they interact with other particles exclusively through the electric force. The muons have a mass which is 206.768 times larger than the mass of the electron, but they have the same elementary electric charge as the electron or positron and the same spin. Scattering experiments tell that the $\mu^{\pm}$ mesons are point particles with a size $< 10^{-16}\,\mathrm{cm}$, just as the electron. In other words, the muons have the same characteristics as the electrons and positrons but for a mass which is about 200 times larger. Consequently the muon is often referred to as a "heavy" electron. If a non-electromagnetic mass is required to explain the mass of the electron then a non-electromagnetic mass 200 times as large as in the electron is required to explain the mass of the muons. These non-electromagnetic masses must be *non-interacting*, otherwise scattering experiments could not find the size of either the electron or the muon at $10^{-16}\,\mathrm{cm}$.

We have explained the mass of the muons in Section 9. According to our model the muons consist of an elementary electric charge and an oscillating lattice of neutrinos. Neutrinos are the only non-interacting matter we know of. In the muon lattice are, according to our model, $(\mathrm{N}-1)/4 = \mathrm{N}'/4$ muon neutrinos ν_μ (respectively anti-muon neutrinos $\bar{\nu}_\mu$), $\mathrm{N}'/4$ electron neutrinos ν_e and the same number of anti-electron neutrinos $\bar{\nu}_e$, one elementary electric charge and the energy of the lattice oscillations. The letter N stands for the number of all neutrinos and antineutrinos in the cubic lattice of the $\pi^{\pm}$ mesons, $\mathrm{N} = 2.854{\cdot}10^9$, Eq.(15). It is, as explained in Section 9, a necessary consequence of the composition of $\pi^{\pm}$ and the decay sequence $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ and $\mu^- \rightarrow \mathrm{e}^- + \bar{\nu}_e + \nu_\mu$, that *there must be* $\mathrm{N}'/4$ *electron neutrinos* ν_e *in the emitted electron*. The explanation of the $\pi^{\pm}$ mesons led to the explanation of the $\mu^{\pm}$ mesons and now leads to the explanation of the mass of $\mathrm{e}^{\pm}$. For the mass of the electron neutrinos and anti-electron neutrinos we found in Eq.(67) that $\mathrm{m}(\nu_e) = \mathrm{m}(\bar{\nu}_e) = 0.365\,\mathrm{milli\text{-}eV/c^2}$. The sum of the energies in the rest masses of the $\mathrm{N}'/4$ neutrinos or antineutrinos in the lattice of the

electron or positron is then

$$\sum \mathrm{m}(\nu_\mathrm{e})\mathrm{c}^2 = \mathrm{N}'/4 \cdot \mathrm{m}(\nu_e)\mathrm{c}^2 = 0.260\,43\,\mathrm{MeV} = 0.5096\,\mathrm{m}(\mathrm{e}^\pm)\mathrm{c}^2\,. \qquad (77)$$

In other words, *1/2 of the rest mass of the electron is approximately equal to the sum of the rest masses of the neutrinos in the electron.* In modern parlance this is the “bare” part of the electron. The bare part is not observable. The other half of the rest mass of the electron must originate from the energy in the electric charge carried by the electron.

From pair production γ + M $\rightarrow$ e^- + e^+ + M, (M being any nucleus), and from conservation of neutrino numbers follows necessarily that there must also be a neutrino lattice composed of $\mathrm{N}'/4$ anti-electron neutrinos, which make up the lattice of the positrons, which lattice has, since $\mathrm{m}(\nu_e) = \mathrm{m}(\bar{\nu}_e)$, the same rest mass as the neutrino lattice of the electron, as it must be for the antiparticle of the electron. Conservation of charge requires conservation of neutrino numbers.

Fourier analysis dictates that a continuum of high frequencies must be in the electrons or positrons created by pair production in a timespan of 10^{-23} seconds. We will now determine the energy $\mathrm{E}_\nu(\mathrm{e}^\pm)$ in the oscillations in the interior of the electron. Since we want to explain the *rest mass* of the electron we can only consider the frequencies of non-progressive waves, either standing waves or circular waves. The sum of the energies of the lattice oscillations is, in the case of the $\pi^\pm$ mesons, given by

$$\mathrm{E}_\nu(\pi^\pm) = \frac{\mathrm{N}\mathrm{h}\nu_0}{2\pi(\mathrm{e}^{\mathrm{h}\nu/\mathrm{kT}}-1)} \int\limits_{-\pi}^{\pi} \phi\, d\phi\,. \qquad (78)$$

This is Eq.(20) which was used to determine the oscillation energy in the π^0 and $\pi^\pm$ mesons. This type of equation was introduced by Born and v. Karman [14] in order to explain the internal energy of cubic crystals. If we apply Eq.(78) to the oscillations in the electron which has $\mathrm{N}'/4$ electron neutrinos ν_e we arrive at $\mathrm{E}_\nu(\mathrm{e}^\pm) = 1/4{\cdot}\mathrm{E}_\nu(\pi^\pm)$, which is mistaken because $\mathrm{E}_\nu(\pi^\pm) \approx \mathrm{m}(\pi^\pm)\mathrm{c}^2/2$ and $\mathrm{m}(\pi^\pm) = 273\,\mathrm{m}(\mathrm{e}^\pm)$. Eq.(78) must be modified in order to be suitable for the oscillations in the electron. It turns out that we must use

$$\mathrm{E}_\nu(\mathrm{e}^\pm) = \frac{\mathrm{N}\mathrm{h}\nu_0 \cdot \alpha_f}{2\pi(\mathrm{e}^{\mathrm{h}\nu/\mathrm{kT}}-1)} \int\limits_{-\pi}^{\pi} \phi\, d\phi\,, \qquad (79)$$

where α_f is the fine structure constant. As is well-known the fine structure constant α_f characterizes the strength of the electromagnetic forces. The

appearance of α_f in Eq.(79) means that the nature of the oscillations in the electron is different from the oscillations in the π^0 or $\pi^\pm$ lattices. With $\alpha_f = \mathrm{e}^2/\hbar\mathrm{c}$ and $\nu_0 = \mathrm{c}/2\pi a$ we have

$$\mathrm{h}\nu_0\alpha_f = \mathrm{e}^2/a\,, \tag{80}$$

which shows that the oscillations in the electron are *electric oscillations*. The appearance of e^2 in Eq.(80) guarantees that the oscillation energy of the electron and positron are the same, as it must be.

There must be $\mathrm{N}'/2$ oscillations of the elements of the electric charge in $\mathrm{e}^\pm$, because we deal with non-progressive waves, which are the superposition of two waves. That means that $2{\times}\mathrm{N}'/4 \cong \mathrm{N}/2$ oscillations are in Eq.(79). As we will see later the spin requires that the oscillations are circular. From Eqs.(78,79) then follows that

$$\mathrm{E}_\nu(\mathrm{e}^\pm) = \alpha_f/2 \cdot \mathrm{E}_\nu(\pi^\pm)\,. \tag{81}$$

$\mathrm{E}_\nu(\pi^\pm)$ is the oscillation energy in the $\pi^\pm$ mesons which can be calculated with Eq.(78). According to Eq.(32) it is

$$\mathrm{E}_\nu(\pi^\pm) = 67.82\,\mathrm{MeV} = 0.486\,\mathrm{m}(\pi^\pm)\mathrm{c}^2 \approx \mathrm{m}(\pi^\pm)\mathrm{c}^2/2\,. \tag{82}$$

With $\mathrm{E}_\nu(\pi^\pm) \approx \mathrm{m}(\pi^\pm)\mathrm{c}^2/2 = 139.57/2\,\mathrm{MeV}$ and $\alpha_f = 1/137.036$ follows from Eq.(81) that the oscillation energy of the electron or positron is

$$\mathrm{E}_\nu(\mathrm{e}^\pm) = \frac{\alpha_f}{2} \cdot \frac{\mathrm{m}(\pi^\pm)\mathrm{c}^2}{2} = 0.254\,623\,\mathrm{MeV} = 0.996\,570\,\mathrm{m}(\mathrm{e}^\pm)\mathrm{c}^2/2\,. \tag{83}$$

If we replace in Eq.(83) the experimental value for $\mathrm{m}(\pi^\pm)$ by the good empirical approximation $\mathrm{m}(\pi^\pm) \cong \mathrm{m}(\mathrm{e}^\pm)(2/\alpha_f)$, Eq.(102), then it follows that

$$\mathrm{E}_\nu(\mathrm{e}^\pm) \cong 1/2 \cdot \mathrm{m}(\mathrm{e}^\pm)\mathrm{c}^2\,. \tag{84}$$

In other words, *1/2 of the energy in the rest mass of the electron is made up by the oscillation energy in* $\mathrm{e}^\pm$. This equation corresponds to Eq.(34) for the oscillation energy in the $\pi^\pm$ mesons.

In Eq.(83) we have determined the value of the oscillation energy in $\mathrm{e}^\pm$ from the product of the very accurately known fine structure constant and the very accurately known rest mass of the $\pi^\pm$ mesons which establishes a firm value of the oscillation energy of $\mathrm{e}^\pm$. The other half of the energy in

m(e$^{\pm}$) is in the rest masses of the neutrinos in the electron, Eq.(77). We can confirm Eq.(83) without using $E_\nu(\pi^\pm)$ with the formula for the oscillation energy in the form of Eq.(88) with N/2 = 1.427·10^{9}, e = 4.803·10^{-10} esu, a = 1·10^{-16} cm, f(T) = 1/1.305·10^{13} and, with the integral being π^2, we obtain $E_\nu(e^\pm) = 0.968\, m(e^\pm)c^2/2$. This calculation involves more parameters than in Eq.(83) and is consequently less accurate than Eq.(83).

In a good approximation the oscillation energy of e$^{\pm}$ in Eq.(83) is equal to the sum of the energies in the rest masses of the electron neutrinos in the e$^{\pm}$ lattice in Eq.(77), or $E_\nu(e^\pm) \cong N'/4 \cdot m(\nu_e)c^2 = N'/4 \cdot m(\bar{\nu}_e)c^2$. Since

$$m(e^\pm)c^2 = E_\nu(e^\pm) + \sum m(\nu_e)c^2 = E_\nu(e^\pm) + N'/4 \cdot m(\nu_e)c^2\,, \qquad (85)$$

it follows from Eqs.(83) and (77) that

$$m(e^\pm)c^2(\mathit{theor}) = 0.5151\,\mathrm{MeV} = 1.0079\, m(e^\pm)c^2(\mathit{exp})\,. \qquad (86)$$

The theoretical rest mass of the electron or positron agrees within the accuracy of the parameters N and $m(\nu_e) = m(\bar{\nu}_e)$ with the measured rest mass.

From Eq.(81) follows with $E_\nu(\pi^\pm) \cong m(\pi^\pm)c^2/2$ from Eq.(82) that

$$2E_\nu(e^\pm) = \alpha_f E_\nu(\pi^\pm) \cong \alpha_f m(\pi^\pm)c^2/2 \cong m(e^\pm)c^2\,,$$

or that

$$m(e^\pm)\cdot 2/\alpha_f = 274.072\, m(e^\pm) = 1.0034\, m(\pi^\pm)\,, \qquad (87)$$

whereas the actual ratio of the mass of the $\pi^\pm$ mesons to the mass of the electron is $m(\pi^\pm)/m(e^\pm) = 273.132$ or $0.9966 \cdot 2/\alpha_f$. The ratio of $m(\pi^\pm)/m(e^\pm)$ which follows from Eq.(87) is similar to what we will find in Eq.(102). This is a necessary condition for the validity of our model of the electron.

We have thus shown that the *rest mass of the electron or positron can be explained* by the sum of the rest masses of the electron neutrinos or anti-electron neutrinos in a cubic lattice with N′/4 electron neutrinos or N′/4 anti-electron neutrinos, plus the mass in the sum of the energy of N/2 standing electric oscillations in the lattice, Eq.(83). The one oscillation added to the even numbered N′/4 oscillations is the oscillation at the center of the lattice, Fig. 8. From this model follows, since it deals with a cubic neutrino lattice, that *the electron is not a point particle*. However, since neutrinos are non-interacting their presence will not be detected in electron scattering experiments.

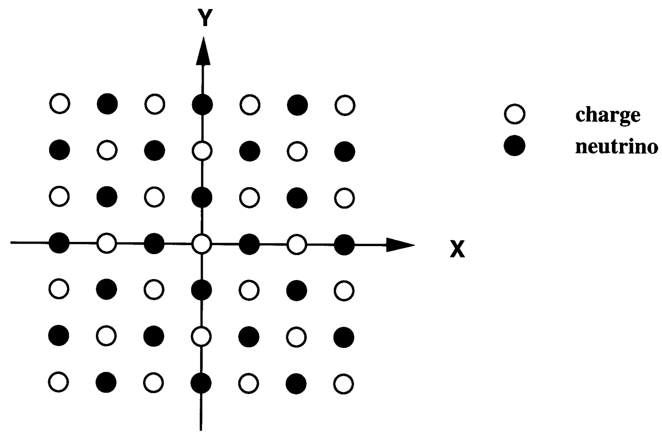

Fig. 8: Horizontal or vertical section through the central part of the electron lattice.

The charge radius of the electron determined by electron scattering experiments is $< 10^{-16}$ cm [65] and seems to contradict the model of the electron proposed here, whose size is on the order of 10^{-13} cm as shown in Appendix D. However, the experimental size does not apply to the circumstances considered here. One has to find the scattering formula for finite size charged cubic lattices and analyze the experimental data with such a scattering formula in order to see whether our model is in contradiction to the experiments. If the electron lattice behaves like a rigid body then electron scattering should take place at the center of the lattice, which is a point [64].

Let us compare the rest mass of the electron to the *rest mass of the muon*, which was explained in Section 9 with an oscillating lattice of muon neutrinos, electron neutrinos and anti-electron neutrinos and an elementary electric charge. The electron has the most simple neutrino lattice, consisting of only one neutrino type, either electron neutrinos or anti-electron neutrinos, and it has the smallest sum of the rest masses of the neutrinos in a particle. The heavy weight of the muon $\mathrm{m}(\mu^\pm) = 206.768\,\mathrm{m}(\mathrm{e}^\pm)$ is a consequence of the heavy weight of the $\mathrm{N}'/4$ muon neutrinos or $\mathrm{N}'/4$ anti-muon neutrinos in the muon lattice. The mass of either a muon neutrino or an anti-muon neutrino is 137 times the mass of an electron or anti-electron neutrino, Eq.(72). That makes the electron neutrinos and anti-electron neutrinos in $\mu^\pm$ as well

as the mass of the electric charge in a first approximation negligible. It then follows from Eq.(98) that $\mathrm{m}(\mu^{\pm})(theor) \cong 3/2 \cdot (\mathrm{m}(\nu_{\mu})/\mathrm{m}(\nu_e)) \cdot \mathrm{m}(\mathrm{e}^{\pm})$ $= 3/2\alpha_f \cdot \mathrm{m}(\mathrm{e}^{\pm}) = 205.554\,\mathrm{m}(\mathrm{e}^{\pm}) = 0.99413\,\mathrm{m}(\mu^{\pm})(exp)$, which proves that the heavy mass of the muons is caused by the heavy $\nu_{\mu}, \bar{\nu}_{\mu}$ neutrinos.

In order to confirm the *validity* of our preceding explanation of the mass of the electron we must show that the sum of the charges in the electric oscillations in the interior of the electron is equal to the elementary electric charge of the electron. We recall that Fourier analysis requires that, after pair production, there must be a continuum of frequencies in the electron and positron. With $\mathrm{h}\nu_0\alpha_f = \mathrm{e}^2/a$ from Eq.(80) follows from Eq.(79) that the oscillation energy in $\mathrm{e}^{\pm}$ is the sum of $2{\times}(\mathrm{N}'/4 + 1) \cong \mathrm{N}/2$ electric oscillations

$$\mathrm{E}_{\nu}(\mathrm{e}^{\pm}) = \frac{\mathrm{N}}{2} \cdot \frac{\mathrm{e}^2}{a} \cdot \frac{f(T)}{2\pi} \int\limits_{-\pi}^{\pi} \phi \, \mathrm{d}\phi \,, \tag{88}$$

with $\mathrm{f(T)} = 1/(\mathrm{e}^{h\nu/kT} - 1) = 1/1.305{\cdot}10^{13}$ from Eq.(21). Inserting the values for N, f(T) and a we find that $\mathrm{E}_{\nu}(\mathrm{e}^{\pm}) = 0.968\,\mathrm{m}(\mathrm{e}^{\pm})\mathrm{c}^2/2 \cong \mathrm{m}(\mathrm{e}^{\pm})\mathrm{c}^2/2$, similar to Eq.(83). The discrepancy between $\mathrm{m}(\mathrm{e}^{\pm})\mathrm{c}^2/2$ and $\mathrm{E}_{\nu}(\mathrm{e}^{\pm})$ so calculated must originate from the uncertainty of the parameters N, f(T) and a in Eq.(88).

In order to determine the charge e in the electric oscillations we replace the integral divided by 2π in Eq.(88), which has the value $\pi/2$, by the sum $\Sigma\, \phi_k \Delta\, \phi$, where k is an integer number with the maximal value $\mathrm{k}_m = (\mathrm{N}/4)^{1/3}$. ϕ_k is equal to $\mathrm{k}\pi/\mathrm{k}_m$ and we have

$$\Sigma\, \phi_k\, \Delta\phi = \sum_{k=1}^{k_m} \frac{k\pi}{k_m} \cdot \frac{1}{k_m} = \frac{k_m(k_m+1)\pi}{2\, k_m^2} \cong \frac{\pi}{2}\,,$$

as it must be. The energy in the individual electric oscillation with index k is then

$$\Delta\mathrm{E}_{\nu}(k) = \phi_k\, \Delta\phi = k\pi/k_m^2\,. \tag{89}$$

Suppose that the energy of the electric oscillations is correctly described by the self-energy of an electric charge Q

$$\mathrm{U} = 1/2\, \cdot\, \mathrm{Q}^2/\mathrm{r}\,. \tag{90}$$

The self-energy of the elementary electric charge is normally used to determine the mass of the electron from its charge, here we use Eq.(90) the other way around, we determine the charge from the energy in the oscillations.

The charge of the electron is contained in the electric oscillations. That means that the electric charge of $e^\pm$ is not concentrated in a point, but is distributed over N/4 = $O(10^9)$ charge elements Q_k. *The charge elements are distributed in a cubic lattice* and the resulting electric field is cubic, not spherical. In the absence of a central force which originates at the center of the particle and affects all parts of the particle the configuration of the particle is not spherical but cubic, just as it was with the shape of the $\pi^\pm$ mesons. For distances large as compared to the sidelength of the cube, (which is $O(10^{-13})$ cm), say at the first Bohr radius which is on the order of 10^{-8} cm, the deviation of the cubic field from the spherical field will be reduced by about 10^{-10}. The charge in all electric oscillations in the electron is

$$\mathrm{Q} = \sum_k \mathrm{Q_k}\,. \tag{91}$$

Setting the radius r in the formula for the self-energy equal to $2\,a$, one charge element is separated from the nearest other by $2a$, we find, with Eqs.(88,89,90), that the charge in the individual electric oscillations is

$$\mathrm{Q_k} = \pm\sqrt{2\pi\,N\,e^2 f(T)/k_m^2}\,\cdot\,\sqrt{k}\,. \tag{92}$$

and with $\mathrm{k}_m = 1/2\cdot(\mathrm{N}/4)^{1/3} = 447$ and

$$\sum_{k=1}^{k_m}\sqrt{k} = 6310.8 \tag{93}$$

follows, after we have doubled the sum over $\sqrt{k}$, because for each index k there is a second oscillation on the negative axis of ϕ, that

$$\mathrm{Q} = \Sigma\,\mathrm{Q_k} = \pm\,5.027\cdot 10^{-10}\ \mathrm{esu}\,, \tag{94}$$

whereas the elementary electric charge is $e^\pm = \pm\,4.803\cdot 10^{-10}$ esu. That means that our theoretical charge of the electron or positron is 1.047 times the elementary electric charge. Within the uncertainty of the parameters the theoretical charge of the electron agrees with the experimental charge e. We have confirmed that it follows from our explanation of the mass of the electron that the electron has, within a 5% error, the correct electric charge.

Each element of the charge distribution is surrounded in the horizontal plane by four electron neutrinos as in Fig. 8, and in vertical direction by an electron neutrino above and also below the element. The electron neutrinos

hold the charge elements in place. We must assume that the charge elements are bound to the neutrinos by the weak nuclear force. The weak nuclear force plays here a role similar to its role in holding, for example, the $\pi^\pm$ or $\mu^\pm$ lattices together. It is not possible, in the absence of a definitive explanation of the neutrinos, to give an explanation for the electro-weak interaction between the electric oscillations and the neutrinos. However, the presence of the range a of the weak nuclear force in e^2/a is a sign that the weak force is involved in the electric oscillations. The attraction of the charge elements by the neutrinos overcomes the Coulomb repulsion of the charge elements. The weak nuclear force is the missing non-electromagnetic force or the Poincaré stress which holds the elementary electric charge together. The same considerations apply for the positive electric charge of the positron, only that then the electric oscillations are all of the positive sign and that they are bound to anti-electron neutrinos.

Finally we learn that Eq.(88) precludes the possibility that the charge of the electron sits only on its surface. The number N in Eq.(88) would then be on the order of 10^6, whereas N must be on the order of 10^9 so that $\mathrm{E}_\nu(\mathrm{e}^\pm)$ can be $\mathrm{m}(\mathrm{e}^\pm)\mathrm{c}^2/2$ as is necessary. In other words, the charge of the electron must be distributed throughout the interior of the electron, as we postulated.

Summing up: The rest mass of the electron or positron originates from the sum of the rest masses of $\mathrm{N}'/4$ electron neutrinos or anti-electron neutrinos in cubic lattices plus the mass in the energy of the electric oscillations in their neutrino lattices. The neutrinos, as well as the electric oscillations, make up 1/2 of the rest mass of $\mathrm{e}^\pm$ each. The electric oscillations are bound to the neutrinos by the weak nuclear force. The sum of the charge elements of the electric oscillations accounts for the elementary charge of the electron, respectively positron. The electron or the positron are not point particles.

One hundred years of sophisticated theoretical work have made it abundantly clear that the electron is not a purely electromagnetic particle. There must be something else in the electron but electric charge, otherwise the electron could not be stable. It is equally clear from the most advanced scattering experiments that the "something else" in the electron must be non-interacting, otherwise it could not be that we find that the charge radius of the electron must be smaller than 10^{-16} cm. The only non-interacting matter we know of with certainty are the neutrinos. So it seems to be natural to ask whether neutrinos are not part of the electron. Actually we did not introduce the neutrinos in an axiomatic manner but rather as a consequence of our explanation of the stable mesons, baryons and muons. It follows nec-

essarily from this model that after the decay of, say, the μ^- meson there must be electron neutrinos in the emitted electron, and that they make up one half of the rest mass of the electron. The other half of the energy in the electron originates from the energy of the electric oscillations. We have thus explained the rest masses of the electron or positron which agree, within 1% accuracy, with the experimental value of $\mathrm{m(e^{\pm})}$. We have learned that the charge of the electron is not concentrated in a single point, but rather is distributed over $\mathrm{O}(10^9)$ elements which are held together with the neutrinos by the weak nuclear force. The sum of the charges in the electric oscillations is, within the accuracy of the parameters, equal to the elementary charge of the electron. From the explanation of the mass and charge of the electron follows, as we will show, the correct spin and magnetic moment of the electron, the other two fundamental properties of the electron. With a cubic lattice of anti-electron neutrinos we also arrive with the same considerations as above at the correct mass, charge, spin and magnetic moment of the positron.

12 The ratios $\mathrm{m}(\mu^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$, $\mathrm{m}(\pi^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$ and $\mathrm{m(p)/m(e)}$

In order to check on the *validity of our explanation of $m(\pi^{\pm})$, $m(\mu^{\pm})$ and also of $m(e^{\pm})$* we will now look at the ratios $\mathrm{m}(\mu^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$, $\mathrm{m}(\pi^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$ and $\mathrm{m(p)/m(e)}$. In order to determine $\mathrm{m}(\mu^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$ we first modify Eq.(69) by setting $\mathrm{N}'/4\cdot\mathrm{m}(\nu_e) = 0.5\,\mathrm{m}(\mathrm{e}^{\pm})$, not at $0.51\,\mathrm{m}(\mathrm{e}^{\pm})$. In other words we say that 1/2 of the mass of the electron is made of neutrinos. If the other half of the mass of the electron originates from the electric charge of the electron, as we have shown in the preceding Section, then the mass of the electron is twice the mass of the sum of the rest masses of the neutrinos in the electron (Eq.69) and we have

$$\mathrm{m}(\mathrm{e}^{\pm}) = \mathrm{N}'/2\cdot\mathrm{m}(\nu_e) \quad \text{or} \quad \mathrm{N}'/2\cdot\mathrm{m}(\bar{\nu}_e)\,. \tag{95}$$

We also set $\mathrm{E}_\nu(\pi^{\pm}) = 0.5\,\mathrm{m}(\pi^{\pm})\mathrm{c}^2$, not at $0.486\,\mathrm{m}(\pi^{\pm})\mathrm{c}^2$ as in Eq.(32). With $\mathrm{E}_\nu(\pi^{\pm}) = \mathrm{E}_\nu(\mu^{\pm})$ from Eq.(63) follows with Eq.(61) and $\mathrm{m}(\pi^{\pm})\mathrm{c}^2 = 2\,\mathrm{E}_\nu(\pi^{\pm})$ that

$$\mathrm{E}_\nu(\mu^{\pm}) = \mathrm{N}'/2\cdot[\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_\mathrm{e})]\mathrm{c}^2\,. \tag{96}$$

From Eq.(66) and with Eqs.(68,71) then follows that

$$\mathrm{m}(\mu^{\pm}) = 3/4\cdot\mathrm{N}'\mathrm{m}(\nu_\mu) + \mathrm{N}'\mathrm{m}(\nu_e)\,, \tag{97}$$

considering only the neutrino masses, not the charge in $\mu^{\pm}$. With $\mathrm{m(e^{\pm})} = \mathrm{N'}/2 \cdot \mathrm{m}(\nu_e)$ from Eq.(95) we have

$$\frac{\mathrm{m}(\mu^{\pm})}{\mathrm{m(e^{\pm})}}(theor) = \frac{3}{2} \cdot \frac{\mathrm{m}(\nu_\mu)}{\mathrm{m}(\nu_e)} + 2\,, \tag{98}$$

or with $\mathrm{m}(\nu_\mu)/\mathrm{m}(\nu_e) \cong 1/\alpha_f$ from Eq.(72) it turns out that

$$\frac{\mathrm{m}(\mu^{\pm})}{\mathrm{m(e^{\pm})}}(theor) \cong \frac{3}{2} \cdot \frac{1}{\alpha_f} + 2 = 207.55 = 1.0038 \cdot \frac{\mathrm{m}(\mu^{\pm})}{\mathrm{m(e^{\pm})}}(exp)\,, \tag{99}$$

with $\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})}(exp) = 206.768$. In order to arrive at the proper ratio of $\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})}$

- *it must be that* $m(\nu_e) = \alpha_f \cdot m(\nu_\mu)$,

as in Eq.(72).

The mass of the muon is, according to Eq.(98), much larger than the mass of the electron because the mass of the muon neutrino is much larger than the mass of the electron neutrino, $\mathrm{m}(\nu_\mu) \cong 137\,\mathrm{m}(\nu_e)$. The ratio of the mass of the muon to the mass of the electron is independent of the number of the neutrinos in both lattices.

Equation (99) is nearly the same as Barut's [51] empirical formula (Eq.59) according to which the muon mass ratio is

$$\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})}(emp) = 3/2\alpha_f + 1 = 206.55 = 0.9989\,\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})}(exp)\,.$$

A much better approximation to the experimental mass ratio is obtained when the $+\,1$ in Barut's formula is replaced by $+\,1.25$. The thus calculated $\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})} = 206.804$ differs then from the measured $\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})} = 206.7683$ by the factor 1.00017.

A better relation between the mass of the muon and the mass of the pion can now be established. According to Eq.(60) it is, empirically,

$$\mathrm{m}(\mu^{\pm})/\mathrm{m}(\pi^{\pm}) \cong 3/4 + \alpha_f\,. \tag{100}$$

With $\alpha_f \mathrm{E}_\nu(\pi^{\pm}) = 2\mathrm{E}_\nu(\mathrm{e}^{\pm})$, (Eq.81), and $\mathrm{m}(\pi^{\pm})\mathrm{c}^2 = 2\mathrm{E}_\nu(\pi^{\pm})$, as well as $\mathrm{m(e^{\pm})c^2} = 2\mathrm{E}_\nu(\mathrm{e}^{\pm})$, follows that

$$\mathrm{m}(\mu^{\pm}) \cong 3/4 \cdot \mathrm{m}(\pi^{\pm}) + 2\,\mathrm{m(e^{\pm})} = 1.00039\,\mathrm{m}(\mu^{\pm})(exp)\,. \tag{101}$$

The reason for the additional term $2\mathrm{m}(\mathrm{e}^\pm)$ on the right hand side of Eq.(101) has still to be found.

Similarly we obtain for the $\pi^\pm$ mesons the ratio

$$\frac{\mathrm{m}(\pi^\pm)}{\mathrm{m}(\mathrm{e}^\pm)}(theor) = 2\,[\frac{\mathrm{m}(\nu_\mu)}{\mathrm{m}(\nu_e)} + 1] \cong \frac{2}{\alpha_f} + 2 = 276.07 = 1.0108\,\frac{\mathrm{m}(\pi^\pm)}{\mathrm{m}(\mathrm{e}^\pm)}(exp)\,, \tag{102}$$

with $\mathrm{m}(\pi^\pm)/\mathrm{m}(\mathrm{e}^\pm)(exp) = 273.1321$. We have, however, only considered the ratio of the rest masses of the neutrinos in $\pi^\pm$ and $\mathrm{e}^\pm$, not the consequences of either the charge or the spin in $\pi^\pm$. Nambu's (improved) empirical formula for the mass ratio $\mathrm{m}(\pi^\pm)/\mathrm{m}(\mathrm{e}^\pm)$ is

$$\frac{\mathrm{m}(\pi^\pm)}{\mathrm{m}(\mathrm{e}^\pm)}(emp) = \frac{2}{\alpha_f}\, -\, 1 = 273.07 = 0.99978\,\frac{\mathrm{m}(\pi^\pm)}{\mathrm{m}(\mathrm{e}^\pm)}(exp)\,. \tag{103}$$

The rest mass of the $\pi^\pm$ mesons is $273 \cong 2 \times 1/\alpha_f$ times larger than the rest mass of the electron because both, the muon neutrino as well as the anti-muon neutrino masses in $\pi^\pm$, are $\cong 137$ times larger than the masses of the electron or anti-electron neutrinos in the electron or positron. If we divide Barut's Eq.(59) for $\mathrm{m}(\mu^\pm)/\mathrm{m}(\mathrm{e}^\pm)$ by Eq.(103) for the ratio $\mathrm{m}(\pi^\pm)/\mathrm{m}(\mathrm{e}^\pm)$, replacing $+1$ in (59) by $+1.25$, we arrive, neglecting a term with α_f^2, at $\mathrm{m}(\mu^\pm)/\mathrm{m}(\pi^\pm) = 3/4 + \alpha_f$, a very good approximation of the experimental mass ratio $\mathrm{m}(\mu^\pm)/\mathrm{m}(\pi^\pm)$ in Eq.(60), as it must be.

For the π^0 meson we find that

$$\mathrm{m}(\pi^0)/\mathrm{m}(\mathrm{e}^\pm)(emp) = 1.000\,268\,(2/\alpha\, -\, 10) = 264.1426\,, \tag{104}$$

as it must be, because $\mathrm{m}(\pi^\pm) - \mathrm{m}(\pi^0) = 0.9988 \cdot 9\mathrm{m}(\mathrm{e}^\pm)$, and because of Eq.(103). The masses of the γ-branch particles are, according to Eq.(1), in a good approximation integer multiples of the mass of the π^0 meson. Their masses are consequently multiples of the right hand side of Eq.(104). For example we find that $\mathrm{m}(\eta)/\mathrm{m}(\mathrm{e}^\pm)(exp) = 1.0144 \cdot 4 \cdot (2/\alpha - 10)$ and we have $\mathrm{m}(\Lambda)/\mathrm{m}(\mathrm{e}^\pm)(exp) = 1.0335 \cdot 8 \cdot (2/\alpha - 10)$.

In order to determine $\mathrm{m}(\mathrm{n})/\mathrm{m}(\mathrm{e}^\pm)$ we start with $\mathrm{K}^0 = (2.)\pi^\pm + \pi^\mp$ and $\mathrm{E}((2.)\pi^\pm) = 4\mathrm{E}_\nu(\pi^\pm) + \mathrm{N}'/2 \cdot [\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_e)]\mathrm{c}^2$, Eq.(35), and with $\mathrm{m}(\pi^\pm) = \mathrm{N}\,[\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_e)]$, Eq.(34). Then $\mathrm{m}(\mathrm{K}^0) = 7\mathrm{N}'/2 \cdot [\mathrm{m}(\nu_\mu) + \mathrm{m}(\nu_e)]$, and with $\mathrm{m}(\mathrm{n}) \cong \mathrm{m}(\mathrm{K}^0 + \overline{\mathrm{K}^0}) = 2\mathrm{m}(\mathrm{K}^0)$ follows that

$$\frac{\mathrm{m}(\mathrm{n})}{\mathrm{m}(\mathrm{e}^\pm)}(theor) = 14\,[\frac{\mathrm{m}(\nu_\mu)}{\mathrm{m}(\nu_e)} + 1] = 14/\alpha_f + 14 = 1932.5 = 1.051\frac{\mathrm{m}(\mathrm{n})}{\mathrm{m}(\mathrm{e}^\pm)}(exp)\,, \tag{105}$$

with m(n)/m(e)(*exp*) = 1838.68. But we have only considered the mass of the neutrino lattice in the neutron, not the consequences of the two positive and two negative elementary charges in the neutron. The empirical value of $\mathrm{m(n)/m(e^{\pm})}$ is $14/\alpha_f - 0.9977 \cdot 80$.

The ratio of the mass of the proton to the mass of the electron, for which an explanation has been looked for since about a hundred years, is m(p)/m(e)(*exp*) = 1836.15. From our theoretical explanation of the neutron follows that

$$\frac{\mathrm{m(p)}}{\mathrm{m(e)}}(theor) \cong 14\,[\frac{\mathrm{m}(\nu_\mu)}{\mathrm{m}(\nu_e)} + 1] - 5/2 = \frac{14}{\alpha_f} + 23/2 = 1930.00\,, \qquad (106)$$

because the energy lost in the β-decay of the neutron is 1.29333 MeV or $1.01239 \cdot 5/2 \cdot \mathrm{m(e^{\pm})\,c^2}$. 1930.0 is 1.051 times the experimental mass ratio 1836.15. We have, again, considered only the mass of the neutrino lattice in the proton, not the consequences of the three elementary electrical charges in the proton. The empirical formula for m(p)/m(e) is

$$\mathrm{m(p)/m(e)}(emp) = 14\,[1/\alpha_f - 6] = 14/\alpha_f - 84 = 0.9901\,\mathrm{m(p)/m(e)}(exp)\,. \qquad (107)$$

The persistent appearance of the fine structure constant α_f in the leading term of the ratio of the masses of the particles to the mass of the electron is a consequence of the preponderance of the mass of the muon neutrinos in the lattices of the particles, following the relation $\mathrm{m}(\nu_e) = \alpha_f \cdot \mathrm{m}(\nu_\mu)$.

We arrive at a better agreement between the theoretical values of $\mathrm{m(e^{\pm})}$, $\mathrm{m}(\mu^{\pm})/\mathrm{m(e^{\pm})}$ and $\mathrm{m}(\pi^{\pm})/\mathrm{m(e^{\pm})}$ and their actual values if we introduce a modified oscillation energy of the electron $\mathrm{E}'_\nu(\mathrm{e^{\pm}})$ given by

$$\mathrm{E}'_\nu(\mathrm{e^{\pm}}) = \mathrm{E}_\nu(\mathrm{e^{\pm}})(1 + \alpha_f/2)\,. \qquad (108)$$

With Eq.(83) we then have

$$\mathrm{E}'_\nu(\mathrm{e^{\pm}}) = 1.000\,206\,\mathrm{m(e^{\pm})c^2}/2\,. \qquad (109)$$

The agreement of $\mathrm{E}'_\nu(\mathrm{e^{\pm}})$ given by Eq.(109) with the actual $\mathrm{m(e^{\pm})c^2}/2$ is an order of magnitude better than it was with Eq.(83).

The modification of the oscillation energy applies only to the electron, not to either $\mu^{\pm}$ or $\pi^{\pm}$, because according to Eq.(83) $\mathrm{E}_\nu(\mathrm{e^{\pm}})$ is proportional to $\mathrm{m}(\pi^{\pm})$ and if $\mathrm{m}(\pi^{\pm})$ were also modified by $(1 + \alpha_f/2)$ then $\mathrm{E}'_\nu(\pi^{\pm})$ would

be proportional to $(1 + \alpha_f/2)^2$ and would be a worse approximation then Eq.(32).

With $\mathrm{m(e^\pm)'}$ standing for the mass of the electron calculated with the modified oscillation energy $\mathrm{E}'_\nu(\mathrm{e}^\pm)$, and assuming that $\mathrm{m(e^\pm)'c^2} = 2\mathrm{E}'_\nu(\mathrm{e}^\pm)$, as seems to be the case according to Eq.(109), we obtain

$$\frac{\mathrm{m}(\mu^\pm)}{\mathrm{m(e^\pm)'}} = \frac{\mathrm{m}(\mu^\pm)}{\mathrm{m(e^\pm)}(1+\alpha_f/2)} \cong \frac{\mathrm{m}(\mu^\pm)}{\mathrm{m(e^\pm)}}(1-\alpha_f/2)\,, \tag{110}$$

and with Eq.(99) we have

$$\frac{\mathrm{m}(\mu^\pm)}{\mathrm{m(e^\pm)'}} = (\frac{3}{2\alpha_f} + 2)(1-\alpha_f/2) = \frac{3}{2\alpha_f} + 1.25\, - \alpha_f\,, \tag{111}$$

which agrees remarkably well with the slightly modified form of Barut's Eq.(59) in which the $+\,1$ is replaced by $+\,1.25$. So we have

$$\frac{\mathrm{m}(\mu^\pm)}{\mathrm{m(e^\pm)'}}(\mathit{theor}) = 206.7967 = 1.000\,137\,\frac{\mathrm{m}(\mu^\pm)}{\mathrm{m(e^\pm)}}(\mathit{exp})\,. \tag{112}$$

Similarly we obtain for $\mathrm{m}(\pi^\pm)/\mathrm{m(e^\pm)'}$ with Eq.(81)

$$\frac{\mathrm{m}(\pi^\pm)}{\mathrm{m(e^\pm)'}} = \frac{2\mathrm{E}_\nu(\pi^\pm)}{2\mathrm{E}'_\nu(\mathrm{e}^\pm)} \cong \frac{2\mathrm{E}_\nu(\pi^\pm)(1-\alpha_f/2)}{\alpha_f/2\cdot 2\mathrm{E}_\nu(\pi^\pm)}\,, \tag{113}$$

or

$$\frac{\mathrm{m}(\pi^\pm)}{\mathrm{m(e^\pm)'}}(\mathit{theor}) = \frac{2}{\alpha_f}(1-\alpha_f/2) = \frac{2}{\alpha_f} - 1 = 273.072\,, \tag{114}$$

whereas the experimental $\mathrm{m}(\pi^\pm)/\mathrm{m(e^\pm)}$ is $273.132 = 1.000\,22\,(2/\alpha_f - 1)$.

Our theoretical calculations of $\mathrm{m}(\pi^\pm)/\mathrm{m(e^\pm)}$ and of $\mathrm{m}(\mu^\pm)/\mathrm{m(e^\pm)}$ agree within the percent range with their experimental values. This can only be if our explanation of $\mathrm{m}(\pi^\pm)$, $\mathrm{m}(\mu^\pm)$ and $\mathrm{m(e^\pm)}$ are correct in the same approximation, and if the ratio $\mathrm{m}(\nu_e) = \alpha_f \mathrm{m}(\nu_\mu)$ (Eq.72) is valid. In other words, our theoretical values of $\mathrm{m}(\pi^\pm)/\mathrm{m(e^\pm)}$ and of $\mathrm{m}(\mu^\pm)/\mathrm{m(e^\pm)}$ *confirm the* validity *of our explanation of $m(\pi^\pm)$, $m(\mu^\pm)$ and of $m(e^\pm)$, as well as the validity of the relation $m(\nu_e) = \alpha_f m(\nu_\mu)$.*

13 The spin of the γ-branch particles

It appears to be crucial for the validity of a model of the elementary particles that the model can also explain the spin of the particles without additional assumptions. The spin or the intrinsic angular momentum is, after the mass and charge, the third most important property of the elementary particles. The standard model does not explain the spin, the spin is imposed on the quarks. As is well-known the spin of the electron was discovered by Uhlenbeck and Goudsmit [59] more than 80 years ago. Later on it was established that the baryons have spin as well, but not the mesons. We have proposed an explanation of the spin of the particles in [67]. For current efforts to understand the spin of the nucleon see Jaffe [68] and of the spin structure of the Λ baryon see Göckeler et al. [69]. Rivas has described his own model of the spin and other spin models in his book [70]. The explanation of the spin requires an unambiguous answer, the spin must be 0 or 1/2 or integer multiples thereof, nothing else. At present we do not have an accepted explanation of the spin.

For the explanation of the spin of the particles it seems to be necessary to have an explanation of the structure of the particles. The spin of a particle is, of course, the sum of the angular momentum vectors of the waves in the particle, plus the sum of the spin vectors of its neutrinos and antineutrinos, plus the spin of the electric charges which the particle carries. It is striking that the particles which consist of a single mode do not have spin, as the $\pi^0, \pi^\pm$ and η mesons do, see Tables 1 and 2. It is also striking that particles whose mass is approximately twice the mass of a smaller particle have spin 1/2 as is the case with the Λ baryon, $\mathrm{m}(\Lambda) \approx 2\mathrm{m}(\eta)$, and with the nucleon $\mathrm{m(n)} \approx 2\mathrm{m}(\mathrm{K}^\pm) \approx 2\mathrm{m}(\mathrm{K}^0)$. The Ξ_c^0 baryon which is a doublet of one mode has also spin 1/2. Composite particles which consist of a doublet of one mode plus one or two other single modes have spin 1/2, as the Σ^0, Ξ^0 and Λ_c^+, Σ_c^0, Ω_c^0 baryons do. The only particle which seems to be the triplet of a single mode, the Ω^- baryon with $\mathrm{m}(\Omega^-) \approx 3\mathrm{m}(\eta)$, has spin 3/2. It appears that the relation between the spin and the modes of the particles is straightforward.

In our model of the γ-branch particles the π^0 and η mesons consist of $\mathrm{N} = 2.85{\cdot}10^9$ standing electromagnetic waves, each with its own frequency. Each of the electromagnetic waves in the π^0 and η mesons may have spin $\mathrm{s} = 1$, because circularly polarized electromagnetic waves have an angular momentum as was first suggested by Poynting [71] and verified by, among others, Allen [72]. The creation of the π^0 meson in the reaction $\gamma\ +\ \mathrm{p} \rightarrow \pi^0\ +$

p and conservation of angular momentum dictates that the sum of the angular momentum vectors of the N electromagnetic waves in the π^0 meson must be zero, $\sum_i j(s_i) = 0$. Either the sum of the spin vectors of the electromagnetic waves in the π^0 meson is zero, or each electromagnetic wave in the π^0 meson has zero spin which would mean that they are linearly polarized. As is well-known, circularly polarized electromagnetic waves have spin s = 1, or an angular momentum. Superposing on such a wave an electromagnetic wave with the same frequency and same amplitude traveling in opposite direction, as is the case for a standing electromagnetic wave, introduces an angular momentum of the opposite direction. In a standing electromagnetic wave the angular momentum of both waves cancel. Linearly polarized electromagnetic waves are not expected to have angular momentum. That this is actually so was proven by Allen [72]. Since the electromagnetic waves in the π^0 and η mesons do not have an angular momentum or since the sum of the spin vectors s_i of the circular electromagnetic waves is zero, the intrinsic angular momentum of the π^0 and η mesons is zero, or

$$j(\pi^0, \eta) = \sum_i j(s_i) = 0 \quad (0 \le i \le \mathrm{N})\,. \tag{115}$$

In this model the π^0 and η mesons do not have an intrinsic angular momentum or spin, as it must be.

We now consider particles such as the Λ baryon whose mass is $\mathrm{m}(\Lambda) = 1.0190 \cdot 2\mathrm{m}(\eta)$. The Λ baryon seems to consist of the superposition of two waves of equal amplitudes and of frequencies ω and $-\omega$, $|-\omega| = \omega$, at each of the N points of the lattice. The waves in such particles must be coupled what we have marked in Tables 1,2 by the $*$ sign. The particles contain then N circular waves, each with its own frequency and each having an angular momentum of $\hbar/2$ as we will see.

The superposition of two perpendicular linearly polarized traveling waves of equal amplitudes and frequencies shifted in phase by $\pi/2$ leads to a circular wave with the constant angular momentum $\mathrm{j} = \hbar$. As is well-known, the total energy of a traveling wave is the sum of the potential and the kinetic energy. In a traveling wave the kinetic energy is always equal to the potential energy. From

$$\mathrm{E}_{pot} + \mathrm{E}_{kin} = \mathrm{E}_{tot} = \hbar\omega\,, \tag{116}$$

follows

$$\mathrm{E}_{tot} = 2\mathrm{E}_{kin} = 2\frac{\Theta\omega^2}{2} = \hbar\omega, \tag{117}$$

with the moment of inertia Θ. It follows that the angular momentum is

$$j = \Theta\omega = \hbar\,. \tag{118}$$

This applies to a traveling wave and corresponds to spin s = 1, or to a circularly polarized photon.

We now add to one monochromatic circular wave with frequency ω a second circular wave with $-\,\omega$ of the same absolute value as ω but shifted in phase by π, having the same amplitude, as we have done in [67]. Negative frequencies are permitted solutions of the equations for the lattice oscillations, Eq.(7). In other words we consider the circular waves

$$x(t) = exp[i\omega t] + exp[-\,i(\omega t + \pi)]\,, \tag{119}$$

$$y(t) = exp[i(\omega t + \pi/2)] + exp[-\,i(\omega t + 3\pi/2)]\ . \tag{120}$$

This can also be written as

$$x(t) = exp[i\omega t] - exp[-i\omega t]\,, \tag{121}$$

$$y(t) = i \cdot (exp[i\omega t] + exp[-i\omega t])\,. \tag{122}$$

If we replace i in the Eqs. above by $-\,i$ we have a circular wave turning in opposite direction.The energy of the superposition of the two waves is the sum of the energies of both individual waves, and since in circular waves E_{kin} = E_{pot} we have with Eq.(117)

$$4\mathrm{E}_{kin} = 4\Theta\omega^2/2 = \mathrm{E}_{tot} = \hbar\omega\,, \tag{123}$$

from which follows that the circular wave has an angular momentum

$$j = \Theta\omega = \hbar/2\,. \tag{124}$$

The superposition of two circular monochromatic waves of equal amplitudes and frequencies ω and $-\,\omega$ satisfies the necessary condition for spin s = 1/2 that the angular momentum is j = $\hbar$/2.

Our model of the particles treats the Λ baryon, which has spin s = 1/2 and a mass m(Λ) = 1.0190 $\cdot$ 2m(η), as the superposition of two particles of the same type with N standing electromagnetic waves. The waves are circular because they are the superposition of two waves with the same absolute

value of the frequency and the same amplitude. The angular momentum vectors of all circular waves in the lattice cancel, except for *the wave at the center of the crystal*. Each oscillation with frequency ω at $\phi > 0$ has at its mirror position $\phi < 0$ a wave with the frequency $-\omega$, which has a negative angular momentum, since j = $\mathrm{mr}^2\omega$ and $\omega = \omega_0\phi$. Consequently the angular momentum vectors of both waves cancel. The center of the lattice oscillates, as all other lattice points do, but with the frequency $\nu(0)$ which is determined by the longest possible wavelength, which is twice the sidelength d of the lattice, so $\nu(0) = \mathrm{c}/2\mathrm{d}$. As the other circular waves in the lattice the circular wave at the center has the angular momentum $\hbar/2$ according to Eq.(124). The angular momentum of the center wave is the only angular momentum which is not canceled by a wave of opposite circulation. There are, as it must be, three possible orientations of the axis of the circular waves.

The net angular momentum of the N circular waves in the lattice reduces to the angular momentum of the center wave and is $\hbar/2$. Since the circular waves in the Λ baryon are the only possible contribution to an angular momentum the intrinsic angular momentum of the Λ baryon is $\hbar/2$ or

$$j(\Lambda) = \sum_i j(\omega_i) = j(\omega_0) = \hbar/2\,. \tag{125}$$

We have thus explained that the Λ and likewise the Ξ_c^0 baryon satisfy the necessary condition that j = $\hbar/2$ for s = 1/2. The intrinsic angular momentum of the Λ baryon is the consequence of the superposition of two circular waves of the same amplitude and the same absolute value of the frequency. Spin 1/2 is caused by the composition of the particles, it is not a contributor to the mass of a particle.

The other particles of the γ-branch, the Σ^0, Ξ^0, Λ_c^+, Σ_c^0 and Ω_c^0 baryons are composites of a baryon with spin 1/2 plus one or two π mesons which do not have spin. Consequently the spin of these particles is 1/2. The spin of all particles of the γ-branch, exempting the spin of the Ω^- baryon, has thus been explained. For an explanation of s($\Sigma^{\pm,0}$) = 1/2, of s($\Xi^{-,0}$) = 1/2 and of s($\Xi_c^{0,+}$) = 1/2, regardless whether the particles are charged or neutral, we refer to [67]. In these cases the elementary electric charge does not seem to be added as an electron or positron to the neutral baryons, but rather through either π^- or π^+ mesons, which do not not have spin. The presence of the $\pi^\pm$ mesons in the charged versions of Σ^0, Ξ^0 and Ξ_c^0 is documented by the appearance of $\pi^\pm$ mesons in the decay products of $\Sigma^\pm$, Ξ^- and Ξ_c^+, whereas in the decays of Σ^0 and Ξ^0 charged mesons do not appear.

14 The spin of the particles of the ν-branch

The characteristic particles of the neutrino-branch are the $\pi^{\pm}$ mesons which have zero spin. At first glance it seems to be odd that the $\pi^{\pm}$ mesons do not have spin, because it seems that the $\pi^{\pm}$ mesons should have spin 1/2 from the spin of the charges $e^{\pm}$ in $\pi^{\pm}$. What happens to the spin of $e^{\pm}$ in $\pi^{\pm}$? The solution of this puzzle is in the composition of the $\pi^{\pm}$ mesons which are, in our model of the particles, made of a lattice of neutrinos and antineutrinos (Fig. 2) each having spin 1/2, the lattice oscillations, and an elementary electric charge.

There is a fundamental difference between the π^0 and $\pi^{\pm}$ mesons. All lattice points in $\pi^{\pm}$ have spin per se, because neutrinos have spin, whereas in the π^0 meson the lattice points do not have spin because they are standing electromagnetic waves. Longitudinal oscillations in the neutrino lattice of the $\pi^{\pm}$ mesons do not cause an angular momentum, $\sum_i j(\nu_i) = 0$, because for longitudinal oscillations $\vec{r} \times \vec{p} = 0$. In the cubic lattice of $N = O(10^9)$ neutrinos and antineutrinos of the $\pi^{\pm}$ mesons the spin of nearly all neutrinos and antineutrinos must cancel because conservation of angular momentum during the creation of the $\pi^{\pm}$ mesons requires that the total angular momentum around a central axis is $\hbar/2$. In fact the spin vectors of all but the neutrino or antineutrino in the center of the lattice cancel. In order for this to be so the spin vector of any particular neutrino in the lattice has to be opposite to the spin vector of the neutrino at its mirror position. As is well-known only left-handed neutrinos and right-handed antineutrinos exist. From $\nu = \nu_0\phi$ (Eq.14) follows that the direction of motion of the neutrinos in e.g. the upper right quadrant ($\phi > 0$) is opposite to the direction of motion in the lower left quadrant ($\phi < 0$). Consequently the spin vectors of all neutrinos or antineutrinos in opposite quadrants are opposite and cancel. The only angular momentum remaining from the spin of the neutrinos of the lattice is the angular momentum of the neutrino or antineutrino at the *center of the lattice* which does not have a mirror particle. Consequently the electrically neutral neutrino lattice consisting of $N'/2$ neutrinos and $N'/2$ antineutrinos and the center particle, each with spin $j(n_i) = 1/2$, has an intrinsic angular momentum $j = \sum_i j(n_i) = j(n_0) = \hbar/2$.

But electrons or positrons added to the neutral neutrino lattice have likewise spin 1/2. If the spin of the electron or positron added to the neutrino lattice is opposite to the spin of the neutrino or antineutrino in the center of

the lattice then the net spin of the π^+ or π^- mesons is zero, or

$$j(\pi^{\pm}) = \sum_i j(\nu_i) + \sum_i j(n_i) + j(\mathrm{e}^{\pm}) = j(n_0) + j(\mathrm{e}^{\pm}) = 0 \quad (0 \leq i \leq \mathrm{N})\,. \tag{126}$$

It is important for the understanding of the structure of the $\pi^{\pm}$ mesons to realize that $\mathrm{s}(\pi^{\pm}) = 0$ can only be explained if the $\pi^{\pm}$ mesons consist of a *neutrino lattice* to which an electron or positron is added whose spin is opposite to the net spin of the neutrino lattice. *Spin 1/2 of the elementary electric charge can only be canceled by something that has also spin 1/2, and in $\pi^{\pm}$ the only conventional choice for that is a single neutrino.*

The spin, the mass and the decay of $\pi^{\pm}$ require that the $\pi^{\pm}$ mesons are made of a cubic neutrino lattice and an elementary electric charge $\mathrm{e}^{\pm}$.

The spin of the $\mathrm{K}^{\pm}$ mesons is zero. With the spin of the $\mathrm{K}^{\pm}$ mesons we encounter the same oddity we have just observed with the spin of the $\pi^{\pm}$ mesons, namely we have a particle which carries an elementary electric charge with spin 1/2, and nevertheless the particle does not have spin. The explanation of $\mathrm{s}(\mathrm{K}^{\pm}) = 0$ follows the same lines as the explanation of the spin of the $\pi^{\pm}$ mesons. In our model the $\mathrm{K}^{\pm}$ mesons are described by the state $(2.)\pi^{\pm} + \pi^0$, that means by the second mode of the lattice oscillations of the $\pi^{\pm}$ mesons plus a π^0 meson. The second mode of the longitudinal oscillations of a neutral neutrino lattice does not have a net intrinsic angular momentum $\sum_i j(\nu_i) = 0$. But the spin of the neutrinos contributes an angular momentum $\hbar/2$, which originates from the neutrino or antineutrino in the center of the lattice, just as it is with the neutrino lattice in the $\pi^{\pm}$ mesons, so $\sum_i j(n_i) = j(n_0) = \hbar/2$. Adding an elementary electric charge with a spin opposite to the net intrinsic angular momentum of the neutrino lattice creates the charged $(2.)\pi^{\pm}$ mode which has zero spin

$$j((2.)\pi^{\pm}) = \sum_i j(n_i) + j(\mathrm{e}^{\pm}) = j(n_0) + j(\mathrm{e}^{\pm}) = 0\,. \tag{127}$$

As discussed in Section 6 it is necessary to add a π^0 meson to the second mode of the $\pi^{\pm}$ mesons in order to obtain the correct mass and the correct decays of the $\mathrm{K}^{\pm}$ mesons. Since the π^0 meson does not have spin the addition of the π^0 meson does not add to the intrinsic angular momentum of the $\mathrm{K}^{\pm}$ mesons. So $\mathrm{s}(\mathrm{K}^{\pm}) = 0$ as it must be.

The explanation of $\mathrm{s} = 0$ of the K^0 and $\overline{\mathrm{K}^0}$ mesons described by the state $(2.)\pi^{\pm} + \pi^{\mp}$ is different because there is now no electric charge whose spin

could cancel the spin of the neutrino lattice. The longitudinal oscillations of the second mode of the neutrino oscillations of $\pi^{\pm}$ in K^0 as well as of the basic $\pi^{\mp}$ mode do not create an angular momentum, $\sum_i j(\nu_i) = 0$. The sum of the spin vectors of the neutrinos in K^0 and $\overline{\mathrm{K}^0}$ is determined by the neutrinos in the second mode of the $\pi^{\pm}$ mesons, or the $(2.)\pi^{\pm}$ state, and the basic $\pi^{\mp}$ mode, each have $\mathrm{N}'/2$ neutrinos and $\mathrm{N}'/2$ antineutrinos plus a center neutrino or antineutrino, so the number of all neutrinos and antineutrinos in the sum of both states, the K^0,$\overline{\mathrm{K}^0}$ mesons, is 2N. Since the size of the lattice of the $\mathrm{K}^{\pm}$ mesons and the K^0 mesons is the same it follows that two neutrinos are at each lattice point of the K^0 or $\overline{\mathrm{K}^0}$ mesons. We assume that Pauli's exclusion principle applies to neutrinos as well. Consequently each neutrino at each lattice point must share its location with an antineutrino. That means that the contribution of the spin of all neutrinos and antineutrinos to the intrinsic angular momentum of the K^0 meson is zero or $\sum_i j(2n_i) = 0$. The sum of the spin vectors of the two opposite charges in either the K^0 or the $\overline{\mathrm{K}^0}$ mesons, or in the $(2.)\pi^{\pm} + \pi^{\mp}$ state, is also zero. Since neither the lattice oscillations nor the spin of the neutrinos and antineutrinos nor the electric charges contribute an angular momentum

$$j(\mathrm{K}^0) = \sum_i j(\nu_i) + \sum_i j(2n_i) + j(\mathrm{e}^+ + \mathrm{e}^-) = 0\,. \tag{128}$$

The intrinsic angular momentum of the K^0 and $\overline{\mathrm{K}^0}$ mesons is zero, or $\mathrm{s}(\mathrm{K}^0,\overline{\mathrm{K}^0}) = 0$, as it must be. In simple terms since, e.g., the structure of K^0 is $(2.)\pi^{+} + \pi^{-}$, the spin of K^0 is the sum of the spin of $(2.)\pi^{+}$ and of π^{-}, both of which do not have spin. It does not seem possible to arrive at $\mathrm{s}(\mathrm{K}^0,\overline{\mathrm{K}^0}) = 0$ if both particles do not contain the N pairs of neutrinos and antineutrinos required by the $(2.)\pi^{\pm} + \pi^{\mp}$ state which we have suggested in Section 6.

In the case of the neutron one must wonder how it comes about that a particle which seems to be the superposition of two particles without spin ends up with spin 1/2. The neutron, which has a mass $\approx 2\mathrm{m}(\mathrm{K}^{\pm})$ or $2\mathrm{m}(\mathrm{K}^0)$, is either the superposition of a K^+ and a K^- meson or of a K^0 and a $\overline{\mathrm{K}^0}$ meson. The intrinsic angular momentum of the superposition of K^+ and K^- is either 0 or $\hbar$, which means that the neutron cannot be the superposition of K^+ and K^-. For a proof of this statement we refer to [67].

On the other hand the neutron can be the superposition of a K^0 and a $\overline{\mathrm{K}^0}$ meson. A significant change in the lattice occurs when a K^0 and a $\overline{\mathrm{K}^0}$ meson are superposed. Since each K^0 meson contains N neutrinos and N antineutrinos, as we discussed in context with the spin of K^0, the number of

all neutrinos and antineutrinos in superposed K^0 and $\overline{K^0}$ lattices is 4N. Since the size of the lattice of the proton as well of the neutron is the same as the size of K^0 each of the N lattice points of the neutron now contains four neutrinos, a muon neutrino and an anti-muon neutrino as well as an electron neutrino and an anti-electron neutrino. The $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ quadrupoles oscillate just like individual neutrinos do because we learned from Eq.(7) that the ratios of the oscillation frequencies are independent of the mass as well as of the interaction constant between the lattice points. In the neutrino quadrupoles the spin of the neutrinos and antineutrinos cancels, $\sum_i j(4n_i) = 0$. The superposition of two circular neutrino lattice oscillations, that means the circular oscillations of frequency ω_i, contribute as before the angular momentum of the center circular oscillation, so $\sum_i j(\omega_i) = j(\omega_0) = \hbar/2$. The spin and charge of the four electrical charges $e^+e^-e^+e^-$ hidden in the sum of the K^0 and $\overline{K^0}$ mesons cancel, $j(4e^\pm) = 0$. It follows that the intrinsic angular momentum of a neutron created by the superposition of a K^0 and a $\overline{K^0}$ meson comes from the circular neutrino lattice oscillations only and is

$$j(\mathrm{n}) = \sum_i j(\omega_i) + \sum_i j(4n_i) + j(4\mathrm{e}^\pm) = \sum_i j(\omega_i) = j(\omega_0) = \hbar/2\,, \quad (129)$$

as it must be. In simple terms, the spin of the neutron originates from the superposition of two circular neutrino lattice oscillations with the frequencies ω and $-\omega$ shifted in phase by π at all lattice points. From those only the angular momentum $\hbar/2$ of the oscillation at the center of the lattice remains.

The spin of the proton is 1/2 and is unambiguously defined by the decay of the neutron $\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e$. We have suggested in Section 10 that $3/4 \cdot \mathrm{N}'$ anti-electron neutrinos of the neutrino lattice of the neutron are removed in the β-decay of the neutron and that the other $\mathrm{N}'/4$ anti-electron neutrinos leave with the emitted electron. The intrinsic angular momentum of the proton originates then from the spin of the central $\nu_\mu\bar{\nu}_\mu\nu_e$ triplet, from the spin of the $e^+e^-e^+$ triplet which is part of the remains of the neutron, and from the angular momentum of the center of the lattice oscillations with the superposition of two circular oscillations. The spin of the central $\nu_\mu\bar{\nu}_\mu\nu_e$ triplet is canceled by the spin of the $e^+e^-e^+$ triplet. The intrinsic angular momentum of the proton is

$$j(\mathrm{p}) = j(\nu_\mu\bar{\nu}_\mu\nu_e)_0 + j(\mathrm{e}^+\mathrm{e}^-\mathrm{e}^+) + j(\omega_0) = j(\omega_0) = \hbar/2\,, \quad (130)$$

as it must be.

The other mesons of the neutrino branch, the $D^{\pm,0}$ and $D_s^{\pm}$ mesons, both having zero spin, are superpositions of a proton and an antineutron of opposite spin, or of their antiparticles, or of a neutron and an antineutron of opposite spin in D^0. The spin of $D^{\pm}$ and D^0 does therefore not pose a new problem.

For an explanation of the spin of $\mu^{\pm}$ we refer to [73]. Since all muon or anti-muon neutrinos have been removed from the $\pi^{\pm}$ lattice in the $\pi^{\pm}$ decay it follows that a neutrino vacancy is at the center of the $\mu^{\pm}$ lattice (Fig. 7). Without a neutrino in the center of the lattice the sum of the spin vectors of all neutrinos in the $\mu^{\pm}$ lattice is zero. However the $\mu^{\pm}$ mesons consist of the neutrino lattice plus an electric charge $e^{\pm}$ whose spin is 1/2. The spin of the $\mu^{\pm}$ mesons originates from the spin of the elementary electric charge carried by the $\mu^{\pm}$ mesons and is consequently $s(\mu^{\pm}) = 1/2$. The same considerations apply for the spin of $\tau^{\pm}$, $s(\tau^{\pm}) = 1/2$.

An explanation of the spin of the mesons and baryons can only be valid if the same explanation also applies to the antiparticles of these particles whose spin is the *same* as that of the ordinary particles. The antiparticles of the γ-branch consist of electromagnetic waves whose frequencies differ from the frequencies of the ordinary particles only by their sign. The angular momentum of the superposition of two circular oscillations with $-\omega$ and ω has the same angular momentum as the superposition of two circular oscillations with frequencies of opposite sign, as in Λ. Consequently the spin of the antiparticles of the γ-branch is the same as the spin of the ordinary particles of the γ-branch. The same considerations apply to the circular neutrino lattice oscillations which cause the spin of the neutron, the only particle of the ν-branch which has spin. In our model of the particles the spin of the neutron and the anti-neutron is the same.

Let us summarize: The spin of the particles of the ν-branch originates from the angular momentum vectors of the lattice oscillations and the spin vectors of the neutrinos in the particles and the spin vector of the elementary electric charge or charges a particle carries. The contribution of all or all but one of the $O(10^9)$ oscillations and $O(10^9)$ neutrinos to the intrinsic angular momentum of the particles must cancel, otherwise the spin cannot be either 0 or 1/2. It requires the symmetry of a cubic lattice for this to happen. The center of the lattices alone determines the intrinsic angular momentum of the oscillations and neutrinos in the lattice. Adding to that the spin vector of one (or more) elementary electric charges with spin 1/2 and we arrive at the total intrinsic angular momentum of a particle. The most illuminating case

are the $\pi^{\pm}$ mesons which do not have spin although they carry an elementary electric charge. Actually the neutrino lattice of the $\pi^{\pm}$ mesons has spin 1/2 from its central neutrino, but this spin vector is canceled by the spin of the electric charge $e^{\pm}$, so $s(\pi^{\pm}) = 0$.

From the foregoing we arrive also at an understanding of the reason for the astonishing fact that the intrinsic angular momentum or spin of the particles is independent of the mass of the particles, as exemplified by the spin $\hbar/2$ of the electron being the same as the spin $\hbar/2$ of the proton, notwithstanding the fact that the mass of the proton is 1836 times larger than the mass of the electron. However, in our model, the spin of the particles including the electron is determined solely by the angular momentum $\hbar/2$ at the center of the lattice, the other angular momentum vectors in the particles cancel. The spin is independent of the number of the lattice points in a cubic lattice. Hence the mass of the particles in the other 10^9 lattice points is inconsequential for the intrinsic angular momentum of the particles. In this model of the particles the spin is independent of the mass of the particles, as it must be.

15 The spin and magnetic moment of the electron

The model of the electron we have proposed in Section 11 has, in order to be valid, to pass a crucial test; the model has to explain satisfactorily the spin and the magnetic moment of the electron. When Uhlenbeck and Goudsmit [59] (U&G) discovered the existence of the spin of the electron they also proposed that the electron has a magnetic moment with a value equal to Bohr's magnetic moment $\mu_B = e\hbar/2m(e^{\pm})c$. Bohr's magnetic moment results from the motion of an electron on a circular orbit around a proton. The magnetic moment of the electron postulated by U&G has been confirmed experimentally, but has been corrected by about 0.11% for the anomalous magnetic moment. If one tries to explain the magnetic moment of the electron with an electric charge moving on a circular orbit around the particle center, analogous to the magnetic moment of hydrogen, one ends up with velocities larger than the velocity of light, which cannot be, as already noted by U&G. It remains to be explained how the magnetic moment of the electron comes about.

We will have to explain the spin of the electron first. The spin, or the intrinsic angular momentum of a particle is, of course, the sum of the angular momentum vectors of all components of the particle. In the electron these are the neutrinos and the electric oscillations. Each neutrino has spin 1/2 and in order for the electron to have s = 1/2 all, or all but one, of the spin vectors of the neutrinos in their lattice must cancel. If the neutrinos are in a simple cubic lattice as in Fig. 8 and the center particle of the lattice is not a neutrino, as in Fig. 8, the spin vectors of all neutrinos cancel, $\Sigma\, j(n_i) = 0$, provided that the spin vectors of the neutrinos of the lattice point in opposite direction at their mirror points in the lattice. Otherwise the spin vectors of the neutrinos would add up and make a very large angular momentum. We follow here the procedure we used in [73] to explain the spin of the muons. The spin vectors of all electron neutrinos in the electron cancel, just as the spin vectors of all muon and electron neutrinos in the muons cancel, because there is a neutrino vacancy at the center of their lattices, Figs. 7,8.

We will now see whether the electric oscillations in the electron contribute to its angular momentum. As we said in context with Eq.(81) there must be two times as many electric oscillations in the electron lattice than there are neutrinos. The oscillations can either be standing waves consisting of two linear oscillations or two circular oscillations with the frequencies ω and $-\,\omega$. Both the standing waves and the circular oscillations must be non-progressive in order to be part of the *rest mass* of a particle. We will now assume that the electric oscillation are circular. Circular oscillations have an angular momentum $\vec{j} = m\,\vec{r} \times \vec{v}$. And, as in the case of the spin vectors of the neutrinos, all or all but one of the O(10^9) angular momentum vectors of the electric oscillations must cancel in order for the electron to have spin 1/2. We describe the superposition of the two oscillations by

$$x(t) = exp[i\omega t] + exp[-\,i(\omega t + \pi)]\,, \tag{131}$$

$$y(t) = exp[i(\omega t + \pi/2)] + exp[-\,i(\omega t + 3\pi/2)]\ , \tag{132}$$

that means by the superposition of an oscillation with the frequency ω and a second oscillation with the frequency $-\,\omega$ as in Eqs.(119,120). The oscillation with $-\,\omega$ is shifted in phase by π. Negative frequencies are permitted solutions of the equations of motion in a cubic lattice, Eqs.(7,14). As is well-known oscillating electric charges should emit radiation. However, this rule

does already not hold in the hydrogen atom, so we will assume that the rule does not hold within the electron either.

In circular oscillations the kinetic energy is always equal to the potential energy and the sum of both is the total energy. From

$$\mathrm{E}_{pot} + \mathrm{E}_{kin} = 2\,\mathrm{E}_{kin} = \mathrm{E}_{tot} \tag{133}$$

follows, as discussed in Eqs.(123,124), that the angular momentum of the superposition of the two circular oscillations is

$$j = \Theta\,\omega = \hbar/2\,. \tag{134}$$

That means that each of the $\mathrm{O}(10^9)$ superposed circular electric oscillations has an angular momentum $\hbar/2$.

The circulation of the circular oscillations in Eqs.(131,132) is opposite for all ϕ of opposite sign. It follows from the equation for the displacements u_n of the lattice points Eq.(5)

$$u_n = Ae^{i(\omega\,t\,+\,n\phi)}\,, \tag{135}$$

that the velocities of the lattice points are given by

$$v_n = \dot{u}_n = i\,\omega_n\,u_n\,. \tag{136}$$

The sign of ω_n changes with the sign of ϕ because the frequencies are given by Eq.(14), that means by

$$\omega_n = \pm\,\omega_0(\,\phi_n + \phi_0\,)\,. \tag{137}$$

Consequently the circulation of the electric oscillations is opposite to the circulation at the mirror points in the lattice and the angular momentum vectors cancel, but for the angular momentum vector of the electric oscillation at the *center of the lattice*. The center circular oscillation has, as all other electric oscillations, the angular momentum $\hbar/2$ as Eq.(134) says. Since the spin vectors n_i of the neutrinos in the electron cancel the angular momentum of the entire electron lattice is

$$j(\mathrm{e}^{\pm}) = \sum_i j(n_i) + \sum_i j(el_i) = j(el_0) = \hbar/2\,, \tag{138}$$

as it must be for spin s = 1/2. The explanation of the spin of the electron given here follows the explanation of the spin of the baryons, as well as the

explanation of the absence of spin in the mesons. A valid explanation of the spin must be applicable to all particles, in particular to the electron, the prototype of a particle with spin.

We will now turn to the magnetic moment of the electron which is known with extraordinary accuracy, $\mu_e = 1.001\,159\,652\,186\,\mu_B$, according to the Review of Particle Physics [2], μ_B being Bohr's magneton. The decimals after $1.00\,\mu_B$ are caused by the anomalous magnetic moment which we will not consider. As is well-known the magnetic dipole moment of a particle with spin is, in Gaussian units, given by

$$\vec{\mu} = g\,\frac{e\hbar}{2mc}\,\vec{s}\,, \tag{139}$$

where g is the dimensionless Landé factor, m the rest mass of the particle that carries the charge e and $\vec{s}$ the spin vector. The g-factor has been introduced in order to bring the magnetic moment of the electron into agreement with the experimental facts. As U&G postulated and as has been confirmed experimentally the g-factor of the electron is 2. With the spin s = 1/2 and g = 2 the magnetic dipole moment of the electron is then

$$\mu_e = e\hbar/2m(e^{\pm})c\,, \tag{140}$$

or one Bohr magneton μ_B in agreement with the experiments, neglecting the anomalous moment. For a structureless point particle Dirac [60] has explained why g = 2 for the electron. However we consider here an electron with a finite size and which is at rest. When it is at rest the electron has still its magnetic moment. Dirac's theory does therefore not apply here. In order to arrive at an explanation of the magnetic moment of the electron it will be necessary to understand the structure of the electron and the origin of the spin.

The only part of Eq.(140) that can be changed in order to explain the g-factor of an electron with structure is the ratio e/m which deals with the spatial distribution of charge and mass. If part of the mass of the electron is non-electromagnetic and the non-electromagnetic part of the mass does not contribute to the magnetic moment of the electron, which to all that we know is true for neutrinos, then the ratio e/m in Eq.(139) is not e/m($e^{\pm}$) in the case of the electron. The elementary charge e certainly remains unchanged, but e/m depends on what fraction of the mass of the electron is of electromagnetic origin and what fraction of the mass is non-electromagnetic.

Only the current, not the mass of a current loop, determines the magnetic moment of a loop. From the very accurately known values of α_f, m($\pi^\pm$)c^2 and m(e$^\pm$)c^2 and from Eq.(83) for the energy in the electric oscillations in the electron $\mathrm{E}_\nu(\mathrm{e}^\pm) = \alpha_f/2 \cdot \mathrm{m}(\pi^\pm)\mathrm{c}^2/2 = 0.996570\,\mathrm{m}(\mathrm{e}^\pm)\mathrm{c}^2/2$ follows that very nearly one half of the mass of the electron is of electric origin, the other half of m(e$^\pm$) is made of neutrinos, Eq.(77), and neutrinos do not contribute to the magnetic moment. That means that in the electron the mass that carries the charge e is approximately m(e$^\pm$)/2. The magnetic moment of the electron is then

$$\vec{\mu}_e = g\frac{e\hbar}{2m(e^\pm)/2 \cdot c}\vec{s}\,, \tag{141}$$

and with s = 1/2 we have $\mu_e = \mathrm{g}\, e\hbar/2m(e^\pm)c$. Because of Eq.(140) the g-factor must be equal to one and is unnecessary. What we have found is akin to what Perkins [23, p.320] states as follows: "The magnetic moment of a charged particle depends on the ratio e/m and thus, classically, for a rotating structure, on the spatial distributions of charge and mass. *If the two distributions are the same, a value g = 1 is obtained on classical arguments.*" (emphasis added).

In other words, if exactly one half of the mass of the electron consists of neutrinos, then it follows automatically that the electron has the correct magnetic moment $\mu_e = e\hbar/2m(e^\pm)c$ without the artificial g-factor. For the explanation of the magnetic moment of the muon see Appendix E.

Conclusions

We conclude that this model of the particles solves a number of problems for which an answer heretofore has been hard to come by. From the creation of the π^0 meson and its decay into $\gamma\gamma$ after 10^{-16} sec follows that the π^0 meson and the other particles of the γ-branch must consist of standing electromagnetic waves. The rest masses of the particles of the γ-branch obey the integer multiple rule. From the creation of the $\pi^\pm$ mesons and their decay follows that the $\pi^\pm$ mesons and the other particles of the ν-branch must consist of a lattice of $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos, their oscillation energies and one or more elementary electric charges. From the explanation of the $\pi^\pm$ mesons follows that the rest mass of the $\mu^\pm$ mesons is $\cong 3/4$ of the rest mass of the $\pi^\pm$ mesons and it also follows that 1/2 of the rest mass of the electron or positron must consist of electron neutrinos or anti-electron neutrinos. The other half of the

rest mass of the electron is in the energy of electric oscillations. The elementary electric charge of the electron is the sum of the charges carried by the individual electric oscillations. The rest mass of the $\mu^{\pm}$ mesons is so much larger than the rest mass of the electron or positron because the rest mass of the muon neutrinos is so much larger than the rest mass of the electron or anti-electron neutrinos. We can determine the strength of the weak force which holds the nuclear lattice together, and explain the strong force between two elementary particles. We have verified the validity of our explanation of $\mathrm{m}(\mathrm{e}^{\pm})$, $\mathrm{m}(\mu^{\pm})$ and $\mathrm{m}(\pi^{\pm})$ and of the relation $\mathrm{m}(\nu_e) = \alpha_f \cdot \mathrm{m}(\nu_\mu)$ by showing that the calculated ratios $\mathrm{m}(\mu^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$ and $\mathrm{m}(\pi^{\pm})/\mathrm{m}(\mathrm{e}^{\pm})$ agree within 1% with the experimental values of these mass ratios. Only photons, neutrinos, charge and the weak nuclear force are needed to explain the masses of the electron, the muon and of the stable mesons and baryons and their antiparticles. We can explain the spin of the baryons and the absence of spin in the mesons, and the spin of the electron and muon as well; without making additional assumptions. We have also determined the rest masses of the electron neutrino, the muon neutrino and the tau neutrino and found that the mass of the electron neutrino is equal to the mass of the muon neutrino times the fine structure constant. Our model is based on the pioneering studies of lattice theory by Born and his students, Born and v. Karman, Born and Landé and Born and Stern.

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Appendix A

The integer multiple rule

When the integer multiple rule was first proposed [12] we relied on the data available at that time, Barnett et al. Rev.Mod.Phys. **68**,611,1996. Since then the data on the particle masses have multiplied and become more precise. It seems now that particles with masses even $70 \times \mathrm{m}(\pi^0)$ follow the integer multiple rule. A list of the particles which obey the integer multiple rule is given in the following Table 3.

Table 3: The particles following the integer multiple rule

	J	m/m(π^0)	multiples		J	m/m(π^0)	multiples
π^0	0	1.0000	$1.0000 \cdot 1\pi^0$	η_c(1S)	0	22.0809	$1.0037 \cdot 22\pi^0$
η	0	4.0563	$1.0141 \cdot 4\pi^0$	J/ψ	1	22.9441	$0.9976 \cdot 23\pi^0$
η'(958)	0	7.0959	$1.0137 \cdot 7\pi^0$	χ_{c0}(1P)	0	25.2989	$1.0119 \cdot 25\pi^0$
η(1295)	0	9.5868	$0.9587 \cdot 10\pi^0$	χ_{c1}(1P)	1	26.0094	$1.0004 \cdot 26\pi^0$
η(1405)	0	10.445	$1.0445 \cdot 10\pi^0$	η_c(2S)	0	26.9528	$0.9983 \cdot 27\pi^0$
η(1475)	0	10.9352	$0.9941 \cdot 11\pi^0$	ψ(2S)	1	27.3091	$1.0115 \cdot 27\pi^0$
				ψ(3770)	1	27.9389	$0.9978 \cdot 28\pi^0$
Λ	1/2	8.2658	$1.0332 \cdot 8\pi^0$	ψ(4040)	1	29.9237	$0.9975 \cdot 30\pi^0$
Λ(1405)	1/2	10.4166	$1.0417 \cdot 10\pi^0$	ψ(4191)	1	31.0543	$1.00175 \cdot 31\pi^0$
Λ(1670)	1/2	12.3725	$1.0310 \cdot 12\pi^0$	ψ(4415)	1	32.7538	$0.9925 \cdot 33\pi^0$
Λ(1800)	1/2	13.335	$1.0258 \cdot 13\pi^0$				
Σ^0	1/2	8.8359	$0.9818 \cdot 9\pi^0$	Υ(1S)	1	70.0884	$1.0013 \cdot 70\pi^0$
Σ(1660)	1/2	12.2984	$1.0249 \cdot 12\pi^0$	χ_{b0}(1P)	0	73.0455	$1.0006 \cdot 73\pi^0$
Σ(1750)	1/2	12.9652	$0.9973 \cdot 13\pi^0$	χ_{b1}(1P)	1	73.2926	$1.0040 \cdot 73\pi^0$
Ξ^0	1/2	9.7412	$0.9741 \cdot 10\pi^0$	Υ(2S)	1	75.8102	$0.9975 \cdot 76\pi^0$
Ω^-	3/2	12.3907	$1.0326 \cdot 12\pi^0$	χ_{b0}(2P)	0	75.8094	$0.9975 \cdot 76\pi^0$
Λ_c^+	1/2	16.9397	$0.9965 \cdot 17\pi^0$	χ_{b1}(2P)	1	75.9795	$0.9997 \cdot 76\pi^0$
$\Lambda_c(2593)^+$	1/2	19.2285	$1.0120 \cdot 19\pi^0$	Υ(3S)	1	76.7185	$0.9963 \cdot 77\pi^0$
$\Sigma_c(2455)^0$	1/2	18.1792	$1.00995 \cdot 18\pi^0$	Υ(4S)	1	78.3795	$1.0049 \cdot 78\pi^0$
Ξ_c^0	1/2	18.3069	$1.01705 \cdot 18\pi^0$	Υ(10860)	1	80.4954	$1.0062 \cdot 80\pi^0$
$\Xi_c'^0$	1/2	19.0996	$1.0052 \cdot 19\pi^0$	Υ(11020)	1	81.6364	$0.9956 \cdot 82\pi^0$
Ξ_c(2790)	1/2	20.6843	$0.9850 \cdot 21\pi^0$				
Ω_c^0	1/2	19.9849	$0.9992 \cdot 20\pi^0$				

The masses are taken from the Review of Particle Physics [2]. Only particles, stable or unstable, with $J \leq 1$ are listed. To be on the safe side, we use only particles which the Particle Data Group considers to be "established". The Ω^- baryon with $J = 3/2$ is also given for comparison, but is not included in the least square analysis. In all there are 41 particles which follow the integer multiple rule. The data of Table 3 are plotted in Fig. 1, the line on Fig. 1 is determined by Eq.(2).

Palazzi [73] has studied the correlation of the masses of the different η mesons using a mass unit of 33.86 MeV/c^2, which is $1.0034\,\mathrm{m}(\pi^0)/4$. In his Fig. 2a he shows that the masses of the η mesons are integer multiples of his mass unit, having a nearly perfect correlation coefficient. This agrees with our integer multiple rule. For the mass of $\psi(4160)$ in Table 3 we have used a recent measurement of Ablikim et al. [74] which places $\mathrm{m}(\psi(4160))$ at 4191.6 MeV/c^2, which is $1.00175 \cdot 31\mathrm{m}(\pi^0)$.

Appendix B

The lattice constant

An equation for the lattice constant in cubic lattices has been given by Born and Landé [41], Eq.(6) therein. Suppose there are N lattice points in a diatomic cubic crystal. There are then N/2 masses m_1 and N/2 masses m_2 and the mass in the crystal is $\mathrm{N}/2\cdot(\mathrm{m}_1 + \mathrm{m}_2)$. In each cell are eight neighboring particles, i.e. there are N/8 such cells. The volume of each cubic cell is a^3 and the total volume of the crystal is $\mathrm{N}a^3/8$. The density ρ is then $\rho = \mathrm{N}/2\cdot(\mathrm{m}_1 + \mathrm{m}_2)/(\mathrm{N}a^3/8)$, from which follows that the lattice constant is given by

$$a^3 = 4\,(\mathrm{m}_1 + \mathrm{m}_2)/\rho\,. \tag{142}$$

We determine the lattice constant of the neutrino lattice with $\mathrm{m}(\nu_e) = 0.365$ milli-eV/c^2 and $\mathrm{m}(\nu_\mu) = 49.91$ milli-eV/c^2, from Eqs.(67,70), and with the density of the $\pi^\pm$ mesons $\rho(\pi^\pm) = \mathrm{m}(\pi^\pm)/\mathrm{Vol}(\pi^\pm)$ or with $\rho = 139.57\,\mathrm{MeV/c^2Vol}(\pi^\pm)$. The volume of the $\pi^\pm$ mesons can be determined from the measured radius of the $\pi^\pm$ mesons, $\mathrm{r}_\pi = 0.880\cdot10^{-13}\,\mathrm{cm} = \mathrm{r}_p$, from Eq.(16). The third power of the lattice constant of the neutrino lattice of the $\pi^\pm$ mesons is then

$$a^3 = \frac{4\,(49.91 + 0.365)\,\text{milli-eV}}{139.57\,\mathrm{MeV/Vol}(\pi^\pm)} = \frac{201.1\,\text{milli-eV}}{139.57\,\mathrm{MeV}} \times \mathrm{Vol}(\pi^\pm) \tag{143}$$

or

$$a^3 = 1.44085 \cdot 10^{-9} \cdot \mathrm{Vol}(\pi^{\pm}) \tag{144}$$

The volume of the $\pi^{\pm}$ mesons is $4\,\pi/3 \cdot \mathrm{r}_{\pi}^3$ with $\mathrm{r}_{\pi} = 0.88 \cdot 10^{-13}\,\mathrm{cm}$, Eq.(16). The measured r_{π} is, however, not equal to the sidelength d of the cubic lattice, which is an integer multiple of the lattice distance a. We must therefore replace the measured r_{π} by $\mathrm{d}/\sqrt[3]{4\pi/3}$. The volume of the cubic $\pi^{\pm}$ meson is then equal to $(\mathrm{r}_{\pi}(\exp))^3$ and it follows that

$$a = 0.9939 \cdot 10^{-16}\,\mathrm{cm}\,. \tag{145}$$

This agrees qualitatively with the neutrino lattice constant we use $a = 1 \cdot 10^{-16}\,\mathrm{cm}$, which we postulated in Eq.(8), the difference with $a = 1{\cdot}10^{-16}\,\mathrm{cm}$ is well within the uncertainty of r_{π}. This agreement is, of course, a consequence of our determination of the neutrino masses $\mathrm{m}(\nu_e)$ and $\mathrm{m}(\nu_{\mu})$ with the help of our postulated a. It is useful to know that lattice theory, expressed by Eq.(6) of B&L or by Eq.(142), leads to the neutrino lattice constant. If the neutrino masses could be determined independently from our calculations, the lattice constant of a neutrino lattice could be calculated from Eq.(142) without making an assumption about a.

Appendix C

Four types of electrons

How come that e^- can carry $N'/4$ electron neutrinos
as well as *$N'/4$ anti-electron neutrinos* ?

Since the rest mass and the spin of ν_e is the same as that of $\bar{\nu}_e$
the most obvious difference between ν_e and $\bar{\nu}_e$ is the

direction of the spin vector.

In $\gamma \to \mathrm{e}^-\mathrm{e}^+$ the spin of e^- and e^+ must be the same. Conservation of angular momentum $(\mathrm{s}(\gamma) = 1)$ requires that the spin vectors of e^- and e^+ point in the same direction. From conservation of neutrino numbers follows that in e^- the same number $\mathrm{N}'/4$ of electron neutrinos must be as anti-electron neutrinos are in e^+. From the explanation of the electron in Section

11, in particular from Fig. 8, we learned that the electron neutrinos or anti-electron neutrinos in $e^{\pm}$ do not contribute to the spin of $e^{\pm}$. That means that the direction of the spin is determined by the direction of the electric field $ef_{\pm}$ in the center of the lattice. The direction of the electric field in the center of the lattice has to have the same direction as the other field elements of the lattice, so that the sum of the electric elements is equal to the elementary electric charge. On the other hand, the sum of the spins of all charge elements of the lattice must cancel, but for the spin of the center element. If no neutrino is in the center of the lattice, the spin of all neutrinos in the lattice cancels.

So the electron with an upward directed spin $s(e^-)\uparrow$ consists of the upward directed electric field $ef_-\uparrow$ and $N'\nu_e/4$ neutrinos.

$$\text{Or}\quad s(e^-)\uparrow = ef_-\uparrow + N'\nu_e/4 \quad\text{and}\quad s(e^-)\downarrow = ef_-\downarrow + N'\bar{\nu}_e/4.$$

The *antiparticles* of $e^-\uparrow$ and $e^-\downarrow$ are

$$s(e^+)\uparrow = ef_+\uparrow + N'\bar{\nu}_e/4 \quad\text{and}\quad s(e^+)\downarrow = ef_+\downarrow + N'\nu_e/4.$$

In the case $\gamma \rightarrow e^-e^+$, when e^- and e^+ have the same spin $\uparrow$,
that would mean that e^- has to be $ef_-\uparrow + N'\nu_e/4$,
and would mean that $e^+\uparrow$ has to be $ef_+\uparrow + N'\bar{\nu}_e/4$.

In the case that e^- and e^+ have spin $\downarrow$ then e^- has to be $ef_-\downarrow + N'\bar{\nu}_e/4$,
and e^+ has to be $ef_+\downarrow + N'\nu_e/4$.

That means that *the neutrino lattice of the electron can consist of either $N'\nu_e/4$ neutrinos or $N'\bar{\nu}_e/4$ antineutrinos.*

Appendix D

The electron radius

As is well-known [2] the classical electron radius is given by

$$r(e)_{cl} = e^2/m(e^{\pm})\,c^2 = 2.817940 \cdot 10^{-13}\,\text{cm}\,. \tag{146}$$

This equation is based on the premise that the electron has a symmetric spherical charge distribution and that the entire mass of m(e) is of electric

origin. As mentioned in Section 11 the electron scattering experiments do not confirm that the electron radius has this value, rather the charge radius of the electron has been found to be on the order of 10^{-16} cm instead of 10^{-13} cm.

In Section 11 we have explained the mass of the electron with a cubic lattice consisting to one half of electric oscillations and to the other half of electron neutrinos, Eqs.(77) and (83). The position of the charge elements of our model of the electron is shown in Fig. 8, they are separated by the distance $2a = 2 \cdot 10^{-16}$ cm. With N/4 charge elements the volume of the cubic charge distribution is $\mathrm{N}/4 \cdot (2a)^3 = 5.708 \cdot 10^{-39}\,\mathrm{cm}^3$, from which follows that the sidelength of the cubic charge distribution of the free electron is

$$\mathrm{d(e)}_{cu} = 1.787 \cdot 10^{-13}\,\mathrm{cm}\,. \tag{147}$$

The volume of the cubic charge distribution of a free electron corresponds to a sphere with the radius

$$\mathrm{r(e)}_{cu} = 1.10866 \cdot 10^{-13}\,\mathrm{cm}\,. \tag{148}$$

For a comparison, it follows from the measured charge radius of the $\pi^{\pm}$ mesons (Eq.16) that

$$\mathrm{r}(\pi^{\pm}) = 0.88 \cdot 10^{-13}\,\mathrm{cm},$$

and that the sidelength of the cubic lattice of the $\pi^{\pm}$ mesons is

$$\mathrm{d}(\pi^{\pm}) = 1.4186 \cdot 10^{-13}\,\mathrm{cm}.$$

The sidelength of a free cubic elementary charge distribution, Eq.(147), is by 25% larger than the sidelength $\mathrm{d}(\pi^{\pm})$ of the $\pi^{\pm}$ mesons. The dimension of the electric charge $\mathrm{e}^{\pm}$ *in* the $\pi^{\pm}$ mesons must be equal to the measured charge radius of $\pi^{\pm}$, which is based solely on the interaction of electrons with $\pi^{\pm}$ mesons, the non-interacting neutrinos in $\pi^{\pm}$ do not contribute to the scattering. The effective radius of a free cubic electron, Eq.(148), is larger than the effective radius $\mathrm{r}(\pi^{\pm})$ of the same charge in the neutrino lattice of a $\pi^{\pm}$ meson. The charge elements in the interior of the $\pi^{\pm}$ mesons are closer together than in the free electron. When the charge $\mathrm{e}^{\pm}$ is introduced into the neutral neutrino lattice of the $\pi^{\pm}$ mesons the electric charge is compressed. That means that a binding energy must be involved when an electron is added to the neutral lattice of a $\pi^{\pm}$ meson. We will pursue this topic in a later paper.

Comparing the classical electron radius Eq.(146) to the effective radius of a free cubic electron, Eq.(148), we find that

$$\mathrm{r(e)}_{cl} = 2.5417\,\mathrm{r(e)}_{cu}\,. \tag{149}$$

The apparent contradiction between our theoretical effective charge radius of the cubic charge distribution of the electron in Eq.(148) and the experimentally measured charge radius of the electron, which is on the order of 10^{-16} cm [65], is a consequence of scattering formulas which do not deal with a rigid cubic charge distribution of a finite size.

Appendix E

The magnetic moment of the muon

The explanation of the magnetic moment of the electron in Section 15 has to pass a critical test, namely it has to be shown that the same considerations lead to a correct explanation of the magnetic moment of the muon $\mu_\mu = \mathrm{e}\hbar/2\mathrm{m}(\mu^\pm)\mathrm{c}$, which is about 1/200th of the magnetic moment of the electron but is known with nearly the same accuracy as μ_e. Both magnetic moments are related through the equation

$$\frac{\mu_\mu}{\mu_e} = \frac{\mathrm{m}(\mathrm{e}^\pm)}{\mathrm{m}(\mu^\pm)} = \frac{1}{206.768}\,, \tag{150}$$

as follows from Eq.(139) applied to the electron and muon. This equation agrees with the experimental results to the sixth decimal. The muon has, as the electron, an anomalous magnetic moment which is too small to be considered here.

As shown in Section 9 the muons consist of a lattice of $\mathrm{N}'/4$ muon neutrinos ν_μ, respectively anti-muon neutrinos $\bar{\nu}_\mu$, of $\mathrm{N}'/4$ electron neutrinos and the same number of anti-electron neutrinos plus an elementary electric charge. For the explanation of the magnetic moment of the muon we follow the same reasoning we have used for the explanation of the magnetic moment of the electron. We say that $\mathrm{m}(\mu^\pm)$ consists of two parts, one part which causes the magnetic moment and another part which does not contribute to the magnetic moment. The part of $\mathrm{m}(\mu^\pm)$ which causes the magnetic moment must contain the electric charge and circular electric oscillations without which there would be no magnetic moment. It becomes immediately clear from the small mass of the electron neutrinos and from Eq.(79) for the energy of the electric oscillations in the electron that $\Sigma\,\mathrm{m}(\nu_e)$ and $\mathrm{E}_\nu(\mathrm{e}^\pm)$ are too small, as compared to the energy in the rest masses of all neutrinos in the

muons, to make up $m(\mu^{\pm})/2$. If, however, the electric charge elements in the muon lattice bind to the muon neutrinos instead to the electron neutrinos, as in the case of the electron, then one obtains $m(\mu^{\pm})/2$ from the sum of the oscillation energy of the muon neutrinos, $(E_{\nu}(\mu^{\pm})/4)$, plus the sum of the energy in the rest masses of the muon neutrinos plus the energy $E_{\nu}(e^{\pm})$ in the electric oscillations

$$\begin{aligned} & 1/4 \cdot E_{\nu}(\mu^{\pm}) \; + \sum m(\nu_{\mu})c^2 + E_{\nu}(e^{\pm}) \\ = \; & 1/4 \cdot E_{\nu}(\mu^{\pm}) + N/4 \cdot m(\nu_{\mu})c^2 + m(e^{\pm})c^2/2 \\ = \; & 53.3125\,\text{MeV} = 0.50457\,m(\mu^{\pm})c^2 \,, \end{aligned} \tag{151}$$

where we have used $E_{\nu}(\pi^{\pm}) = E_{\nu}(\mu^{\pm})$ according to Eq.(63), with $E_{\nu}(\pi^{\pm}) = m(\pi^{\pm})c^2/2 = 69.7851\,\text{MeV}$ and also $E_{\nu}(e^{\pm}) = m(e^{\pm})c^2/2$ from Eq.(84), as well as $m(\nu_{\mu})c^2 = 49.91$ milli-eV, (Eq.70), $m(\mu^{\pm})c^2 = 105.6583\,\text{MeV}$ and $N' \cong N$. Eq.(151) says that the part of $m(\mu^{\pm})$ which carries the electric charge and causes the magnetic moment is $\approx m(\mu^{\pm})/2$, provided that the charge elements bind to the muon neutrinos instead of the electron neutrinos. The remaining part of $m(\mu^{\pm})$ which does not carry charge and does not contribute to the magnetic moment is given by

$$\begin{aligned} & 3/4 \cdot E_{\nu}(\mu^{\pm}) \; + \sum (m(\nu_e) + m(\bar{\nu}_e))c^2 + m(e^{\pm})c^2/2 \\ = \; & 3/4 \cdot E_{\nu}(\mu^{\pm}) + 3/4 \cdot N m(\nu_e)c^2 \\ = \; & 53.1201\,\text{MeV} = 0.50275\,m(\mu^{\pm})c^2 \,. \end{aligned} \tag{152}$$

The additional $m(e^{\pm})c^2/2$ on the top line of Eq.(152) originates from the energy in the mass of the N/4 electron neutrinos which make up the neutral part of the electron. For $m(\nu_e) = m(\bar{\nu}_e)$ we use the value 0.365 milli-eV/c^2 as in Eq.(67). The sum of the mass of the charged part of $m(\mu^{\pm})$ plus the neutral part of $m(\mu^{\pm})$ is $1.0073\,m(\mu^{\pm})$. It is important to note that Eqs.(151,152) depend critically on the validity of $E_{\nu}(\mu^{\pm}) = E_{\nu}(\pi^{\pm})$, from Eq.(63).

If the electric charge elements bind to the muon neutrinos in the muon lattice and if the charged part of the muon makes up 1/2 of the mass of the muon as in Eq.(151) then it follows from Eq.(139) that the magnetic moment of the muon is given by

$$\vec{\mu}_{\mu} = \frac{e\hbar}{2m(\mu^{\pm})/2 \cdot c} \cdot \vec{s}\,. \tag{153}$$

With s = 1/2 we have

$$\mu_\mu = \mathrm{e}\hbar/2\mathrm{m}(\mu^\pm)\mathrm{c} \tag{154}$$

as it must be, without the artificial g-factor.

We have thus shown that we can explain the magnetic moment of the muon with the same concept that we have applied to the explanation of the magnetic moment of the electron, namely that 1/2 of the mass of the electron does not contribute to the magnetic moment because this half of the mass does not carry charge. In the case of the muon the same is true, provided that the charge elements bind to the muon neutrinos in the muon lattice.